Parents' Preferences for Expenditure on ChildrenWhen at Least One Parent Works and Preferences are Non-Separable

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Parents' Preferences for Expenditure on Children When At Least One Parent Works and Preferences Are Non-separable*

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Abstract

The evaluation of policies designed to increase the early investment in children requires knowledge of the parents' preferences for expenditure on children. In this paper it is shown that these preferences can be recovered using variables available in commonly used datasets. The method proposed in this paper improves upon the approach developed by Blundell, Chiappori, and Meghir (2005) in two ways. First, it only requires that one parent supplies a positive amount of labor. In the PSID, 98% of families with children have at least one parent who participates in the labor market. The method proposed by Blundell, Chiappori, and Meghir (2005) requires that both parents work. In the PSID, both parents work only in 64% of families with children. Second, the approach presented in this paper does not require that the parents' preferences are separable in expenditure on children or the availability of a distribution factor.

JEL classification: D10, D91, H41.

Keywords: Public consumption, expenditure on children, collective models, household decisions, preferences.

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1 Introduction

This paper develops a method for recovering the parents' preferences for expenditure on children. The method proposed here only requires that one parent participates in the labor market and it does not require that the parents' preferences are separable between expenditure on children and expenditure on other goods.

A large number of papers have attempted to evaluate the effect of investments in children on future earnings and future cognitive and non-cognitive abilities. This literature is thoroughly reviewed in Cunha, Heckman, Lochner, and Masterov (2005). There are two main messages in this literature. First, differences in family income explain a large portion of the observed differences in test outcomes and future earnings of children. Second, early investments in children have a positive and significant effect on test outcomes and future earnings. The general recommendation is the implementation of public policies that raise the early investment in children living in deprived environments. There are several examples of policies that are designed to achieve this goal and they can be divided into two groups. The first group includes policies that attempt to increase the early investment in children mostly during the time they are in school. The main examples are the Perry Preschool program, the Abecedarian program, and the Chicago Child-Parent Centers program. The second group is composed of policies designed to change the early investment in children while they are at home with their parents. There are at least two examples of such policies: the home visitation program in Elmira, NY, and a similar program in Memphis, TN. A good understanding of the effect of such policies on children outcomes requires two types of information. First, one needs to know how much parents value their children. Second, it is important to know whether there is heterogeneity in such values across income quartiles, which may arise because of selection and matching in the marriage market. If part of the difference in early investments in children is explained by heterogeneity in preferences for expenditure on children, one would expect policies in the first group to be more effective.

The main contribution of this paper is to develop a general approach that enables one to recover the parents' preferences for expenditure on children. This is not the first paper that attempts to identify the preferences for children. In a seminal and insightful paper, Blundell, Chiappori, and Meghir (2005) show that the parents' preferences for expenditure on children can be identified if

the following conditions are satisfied. First, both parents participate in the labor market. Second, the parents' utilities are separable in expenditure on children or there exists one distribution factor, i.e. an exogenous variable that affects the household decision process only through its impact on the individual decision power. The first condition is restrictive if one is interested in families with children. In the Panel Study of Income Dynamics (PSID) both spouses work in only 64% of families with children and parents between the ages of 21 and 60. The second condition is also restrictive. Blundell et al. (2005) recognize that the separability assumption will generally not be satisfied and suggest the use of a distribution factor. The existence of distribution factors is unquestionable. The typical examples are the ratio of women to men in the local marriage market or divorce laws. The main problem with the use of distribution factors is that they are generally aggregate variables. There is therefore little variation across households, whereas the method proposed by Blundell et al. (2005) requires a large amount of cross-sectional variation.

In this paper it is shown that the preferences for expenditure on children can be recovered without the two restrictions required by Blundell et al. (2005). The method proposed here enables one to identify these preferences even if only one parent supplies a positive amount of labor. In the PSID 98% of families with children and parents between the ages of 21 and 60 have at least one spouse with a positive amount of labor hours. Moreover, it is shown that the preferences for expenditure on children can be recovered for non-separable preferences even if no distribution factor is observed or the variation in the available distribution factor is insufficient. To provide the intuition behind the more general result obtained in this paper observe that Blundell et al. (2005) employ a static model in the identification of the preferences for expenditure on children. Thus, only the intratemporal optimality conditions are used to recover the parents' preferences. In this paper, I use an intertemporal generalization of the model considered in Blundell et al. (2005). As a consequence the intertemporal optimality conditions can be used jointly with the intratemporal ones to recover the parents' preferences. The addition of the intertemporal conditions enables one to recover the parents' preferences for children without the two restrictions imposed in Blundell et al. (2005).

The approach considered in this paper has one weakness that is not present in Blundell et al. (2005). Part of the identification relies on Euler equations. In principle, the functions in the Euler equations can be identified non-parametrically. However, a paper with a non-parametric estimator

for Euler equations is not available yet. Consequently, until such paper becomes available, the result presented here relies on parametric methods of identification of Euler equations. From a practical viewpoint this is not an important limitation since almost all empirical works use parametric specifications of preferences. However, from a theoretical viewpoint the method proposed in this paper is less general than the approach proposed in Blundell et al. (2005).[1]

This paper is related to the literature on the collective representation of household behavior. Manser and Brown (1980) and McElroy and Horney (1981) are the first papers to characterize the household as a group of agents making joint decisions. In these papers the household decision process is modeled as a Nash bargaining problem. Chiappori (1988; 1992) extends their analysis to allow for any type of efficient decision process. The model used in the present paper is an intertemporal generalization of Chiappori's static collective model. This paper therefore also contributes to a growing literature which attempts to model and estimate the intertemporal aspects of household decisions using a collective formulation.[2]

The identification of individual preferences has been discussed in other papers. For instance, Fong and Zhang (2001) consider a model in which leisure is partly private consumption and partly public consumption. Under a separability assumption, they show that the preferences for leisure can be identified even if only total leisure is observed. Donni (2004) considers a collective model with private and public consumption, derives testable implications from the model, and shows that some features of the household decision process can be identified.

The paper is organized as follows. Section 2 describes the intertemporal model of the household used in the identification of individual preferences. Section 3 considers a general example of the

[1] I am indebted with Jinyong Hahn for suggesting that in principle Euler equations can be identified using nonparametric methods.

[2] The static collective model has been extensively tested and estimated. Schultz (1990) and Thomas (1990) are two of the first papers to test the static unitary model against the static collective model. Browning, Bourguignon, Chiappori, and Lechene (1994) perform a similar test and estimate the intra-household allocation of resources. Thomas, Contreras, and Frankenberg (1997) have shown, using Indonesian data, that the distribution of wealth by gender at marriage has a significant impact on children's health in places where wealth remains under the control of the initial owner. Chiappori, Fortin, and Lacroix (2002) analyze theoretically and empirically the impact of the marriage market and divorce legislations on household labor supply using a static collective model. Blundell, Chiappori, Magnac, and Meghir (2001) develop and estimate a static collective labor supply framework which allows for censoring and nonparticipation in employment. There are several examples of papers that analyze household intertemporal decisions. Lundberg, Startz, and Stillman (2003) use a collective model with no commitment to explain the consumption-retirement puzzle. Guner and Knowles (2004) simulate a model in which marital formation affects the distribution of wealth in the population. Van der Klaauw and Wolpin (2004) formulate and estimate an efficiency model of retirement and saving decisions of elderly couples. Duflo and Udry (2004) study the resource allocation and insurance within households using data from Côte D'Ivoire. Mazzocco (2004) analyzes the effect of efficient risk sharing on household decisions.

intertemporal model and discusses how the intertemporal optimality conditions help in the identification of the parents' preferences for expenditure on children. Section 4 presents the identification result for a general class of preferences. Section 5 concludes.

2 An Intertemporal Collective Model

This section outlines the model that will be used in the identification of individual preferences. It is a variation of the model developed in Mazzocco (forthcoming), which is a generalization to an intertemporal setting of the static collective model proposed by Chiappori (1988; 1992).

Consider a two-person household living for $\mathcal{T}$ periods in an uncertain environment. In each period $t \in \{0, ..., \mathcal{T}\}$ and state of nature $\omega \in \Omega$, member i receives non-labor income $y^i(t, \omega)$, supplies labor in quantity $h^i(t, \omega)$, and chooses expenditure on a private composite good $c^i(t, \omega)$ and on children $Q(t, \omega)$. Since children are for the most part a public good for their parents, $Q(t, \omega)$ will be modeled as public consumption. Let $C(t, \omega)$ be household total private consumption and let $l^i(t, \omega) = 1 - h^i(t, \omega)$ be leisure of member i, where the time endowment is normalized to 1. The price of private and public consumption will be denoted by $p(t, \omega)$ and $P(t, \omega)$, and agent i's wage by $w_i(t, \omega)$. Household members can save jointly using a risk-free asset. Denote by $s(t, \omega)$ and $R(t)$, respectively, the amount of wealth invested in the risk-free asset and its gross return.[3] Each household member is characterized by individual preferences, which are assumed to be separable over time and across states of nature. The corresponding utility function U^i is assumed to be increasing, concave, and twice continuously differentiable. Agent i's utility function can depend on agent j's private consumption and leisure but only additively, i.e.

$$U^i\left(c^1, c^2, l^1, l^2, Q\right) = u^i\left(c^i, l^i, Q\right) + \delta_i u^j\left(c^j, l^j, Q\right),$$

where δ_i is the altruism parameter. It is assumed that the two spouses have the same discount factor β.[4]

It is assumed that household members make decisions cooperatively, in the sense that every decision is efficient and hence on the ex-ante Pareto frontier. Household behavior can therefore be

[3] The results of the paper are still valid if risky assets are introduced in the model.

[4] This assumption is made for expositional purposes. If the individual discount factors are different, it can be shown that the identification method proposed here still works with small modifications.

characterized as the solution of the following Pareto problem:[5]

$$\max_{\left\{c_t^1,c_t^2,l_t^1,l_t^2,Q_t,s_t\right\}} \mu E_0\left[\sum_{t=0}^{\mathcal{T}}\beta^t u^1(c_t^1,l_t^1,Q_t)\right]+(1-\mu)\,E_0\left[\sum_{t=0}^{\mathcal{T}}\beta^t u^2(c_t^2,l_t^2,Q_t)\right] \tag{1}$$

$$s.t.\ \sum_{i=1}^{2}\left(p_t c_t^i+w_{it}l_t^i\right)+P_tQ_t+s_t=\sum_{i=1}^{2}\left(y_t^i+w_{it}\right)+R_t s_{t-1}\quad\forall t,\omega$$

$$0\le l_t^i\le 1\quad\forall i,t,\omega,\qquad s_T\ge 0\quad\forall\omega,$$

where μ can be interpreted as the decision power of the first household member and it is a combination of Pareto weights and altruism parameters.

To discuss the identification of individual preferences it is helpful to rewrite the household problem using a two-stage formulation. Under the assumption that individual preferences are separable over time and across states of nature, the solution of the household problem (1) is equivalent to the solution of the following two-stage problem. In the second stage, conditional on the amount of resources available in period t and state ω, the household chooses how much to spend on consumption and leisure. Formally, let $\bar{Y}(t,\omega)$ be the amount of resources available in period t and state ω. In the second stage, the household solves the following static problem for each t and ω:

$$V\left(\bar{Y}_t,w_{1t},w_{2t},p_t,P_t\right)=\max_{c_t^1,l_t^1,c_t^2,l_t^2,Q_t}\ \mu u^1\left(c_t^1,l_t^1,Q_t\right)+(1-\mu)\,u^2\left(c_t^2,l_t^2,Q_t\right)$$

$$s.t.\ \sum_{i=1}^{2}\left(p_t c_t^i+w_{it}l_t^i\right)+P_tQ_t=\bar{Y}_t$$

$$0\le l_t^i\le 1\quad\text{for } i=1,2.$$

The standard Marshallian demand functions for public consumption, household private consumption, and leisure can be derived as the solution of this second-stage problem. They depend on the prices of public and private consumption, the individual wages, and the resources available in period t and state ω, i.e. $Q_t=Q\left(p_t,P_t,w_{1t},w_{2t},\bar{Y}_t\right)$, $C_t=c_t^1+c_t^2=C\left(p_t,P_t,w_{1t},w_{2t},\bar{Y}\right)$, $l_t^1=l^1\left(p_t,P_t,w_{1t},w_{2t},\bar{Y}_t\right)$, and $l_t^2=l^2\left(p_t,P_t,w_{1t},w_{2t},\bar{Y}_t\right)$. In the first stage the household chooses the optimal allocation of resources to each period and state of nature by solving the following

[5]Unless required for expositional clarity, the dependence on the states of nature will be suppressed in the rest of the paper.

dynamic problem:

$$\max_{\{\bar{Y}_t, s_t\}} \sum_{t=0}^{\mathcal{T}} E_0 \left[\beta^t V\left(\bar{Y}_t, w_{1t}, w_{2t}, p_t, P_t\right)\right]$$

$$s.t.\ \bar{Y}_t = \sum_{i=1}^{2} \left(y_t^i + w_{it}\right) + R_t s_{t-1} - s_t \quad \forall t, \omega$$

$$s_T \geq 0\ \forall \omega.$$

The two-stage formulation will be used to describe the type of variation required in the identification of the individual preferences for expenditure on children.

3 A General Example

In this section I will consider an example which illustrates how the intertemporal conditions help in the identification of the parents' preferences for expenditure on children. In an attempt to provide a clear intuition of the identification results, in this session I will consider an environment with no uncertainty. I will only discuss the case in which only agent 1, the father, works since in this case the identification of the parameters of interest is more difficult to achieve.

In the example discussed in this section I will consider a specific functional form for the parents' preferences. It is assumed that each parent has a utility function that is non-separable in public consumption and has the following form:

$$U\left(l^i, c^i, Q\right) = \left(c^i\right)^{\sigma_i} \left(l^i\right)^{\theta_i} (Q)^{\gamma_i} + \delta_i \ln Q.$$

The utility function is a standard Cobb-Douglas utility function augmented to include a public good. The public good enters individual preferences in two different ways: through a separable function and through a function in which public consumption is non-separable from leisure and private consumption. This feature of the utility function will enable me to describe which variation is required for the identification of the non-separable part of the preferences for public consumption and which variation is needed to recover the separable part.

The problem of identifying the parameters of interest can be stated in the following way. The econometrician knows public consumption, household private consumption, the father's leisure, the father's wage, the prices of private and public consumption, and the amount of resources that

the household decides to allocate to each period. Since the mother does not work, no variation in her wage and leisure is observed. A dataset in which all these variables are observed is the Consumer Expenditure Survey (CEX). Using these variables, the econometrician can recover non-parametrically the Marshallian demand functions $Q = Q\left(p, P, w_1, \bar{Y}\right)$, $C = C\left(p, P, w_1, \bar{Y}\right)$, and $l^1 = l^1\left(p, P, w_1, \bar{Y}\right)$, which are the solution of the second stage of the household problem. They will therefore be assumed to be known. Note that the Marshallian demand function for the mother's leisure cannot be recovered because by assumption there is no variation in her leisure. Since the Marshallian demand functions are known, the derivatives of public consumption, private consumption, and the father's leisure with respect to wages, resources, and prices are also known. Given this information, the econometrician is interested in recovering the preference parameters and the decision power parameter.

In the parametric examples considered in this section, identification can be easily analyzed using the first order conditions for private consumption, leisure, and public consumption. The following approach will be employed. The first order conditions will be used to derive a set of equations that depend on the parameters of interest and variables that are known. The equations can then be solved for the parameters of interest. If a unique solution exists, the model is identified. If more than one solution exist, the model is not identified.

I will start with the derivation of the first order conditions. Denote by λ_t the multiplier of the budget constraint of the household problem in period t and let $\mu_1 = \mu$ and $\mu_2 = 1 - \mu$. In the example considered here, the first order conditions for private consumption of parent i can be written in the form

$$\beta^t \mu_i \sigma_i \left(c_t^i\right)^{\sigma_i - 1} \left(l_t^i\right)^{\theta_i} \left(Q_t\right)^{\gamma_i} = p_t \lambda_t,$$

the leisure first order condition of the working parent takes the form

$$\beta^t \mu_1 \theta_1 \left(c_t^1\right)^{\sigma_1} \left(l_t^1\right)^{\theta_1 - 1} \left(Q_t\right)^{\gamma_1} = w_{1t} \lambda_t,$$

and the public consumption first order condition can be written as

$$\beta^t \sum_{i=1}^{2} \mu_i \left(\gamma_i \left(c_t^i\right)^{\sigma_i} \left(l_t^i\right)^{\theta_i} \left(Q_t\right)^{\gamma_i - 1} + \frac{\delta_i}{Q_t}\right) = P_t \lambda_t,$$

where $l_t^2 = 1$ for the mother. Finally, in an environment without uncertainty, the first order

condition that captures the optimal allocation of resources over time has the following form:

$$\lambda_t = R_{t+1}\lambda_{t+1}.$$

Using these first order conditions one can derive the five optimality conditions that will be employed in the identification of the parameters of interest: (i) an equation stating that the marginal rate of substitution between consumption and leisure of the working parent must equal his real wage; (ii) the efficiency condition for private consumption; (iii) the efficiency condition for public consumption; (iv) the private consumption Euler equation for the mother; (v) the private consumption Euler equation for the father.[6] In the present example, the first optimality condition has the following form:

$$\frac{\theta_1}{\sigma_1}\frac{c_t^1}{l_t^1} = \frac{w_{1t}}{p_t}.$$

The private consumption efficiency condition can be written as

$$\frac{\sigma_1 \left(c_t^1\right)^{\sigma_1 - 1} \left(l_t^1\right)^{\theta_1} Q_t^{\gamma_1}}{\sigma_2 \left(c_t^2\right)^{\sigma_2 - 1} Q_t^{\gamma_2}} = \frac{1-\mu}{\mu}. \tag{2}$$

Using the first order conditions for public and private consumption, the public consumption efficiency condition can be written as follows:

$$\frac{\gamma_1}{\sigma_1}\frac{c_t^1}{Q_t} + \frac{\delta_1}{\sigma_1}\frac{1}{\left(c_t^1\right)^{\sigma_1 - 1}\left(l_t^1\right)^{\theta_1} Q_t^{\gamma_1 + 1}} + \frac{\gamma_2}{\sigma_2}\frac{c_t^2}{Q_t} + \frac{\delta_2}{\sigma_2}\frac{1}{\left(c_t^2\right)^{\sigma_2 - 1} Q_t^{\gamma_2 + 1}} = \frac{P_t}{p_t}.$$

Finally, the father's private consumption Euler equation takes the form

$$\beta R_{t+1}\frac{p_t}{p_{t+1}}\left(\frac{c_{t+1}^1}{c_t^1}\right)^{\sigma_1 - 1}\left(\frac{l_{t+1}^1}{l_t^1}\right)^{\theta_1}\left(\frac{Q_{t+1}}{Q_t}\right)^{\gamma_1} = 1,$$

whereas the mother's can be written as follows:

$$\beta R_{t+1}\frac{p_t}{p_{t+1}}\left(\frac{c_{t+1}^2}{c_t^2}\right)^{\sigma_2 - 1}\left(\frac{Q_{t+1}}{Q_t}\right)^{\gamma_2} = 1.$$

I will now discuss how the parameters of the non-separable part of the father's preferences σ_1, θ_1, and γ_1 can be recovered by using his private consumption Euler equation. The Euler equations depend on private consumption which is not observed. However, one can use the optimality

[6]The model considered in this paper is over-identified in the sense that the number of optimality conditions that can be used to recover the parameters of interest is greater than the number of parameters. For instance, the public consumption Euler equation could be used in place of one of the five conditions employed in this section. The optimality conditions that are not used in the identification of the parameters can be used to test the model.

condition that relates the marginal rate of substitution of the father to his real wage to derive the father's private consumption as a function of own leisure and own real wage, i.e.

$$c_t^1 = \frac{\sigma_1}{\theta_1}\frac{w_{1t}}{p_t}l_t^1.$$

In each period the sum of individual private consumption must equal total household private consumption C_t, which is observed. As a consequence, the mother's private consumption can also be written as a function of variables that are observed, i.e.

$$c_t^2 = C_t - \frac{\sigma_1}{\theta_1}\frac{w_{1t}}{p_t}l_t^1.$$

The private consumption Euler equation of the father can now be written in terms of variables that are observed by substituting out individual consumption. After the substitution, this intertemporal optimality condition becomes

$$\beta R_{t+1}\left(\frac{p_t}{p_{t+1}}\right)^{\sigma_1}\left(\frac{w_{1t+1}}{w_{1t}}\right)^{\sigma_1-1}\left(\frac{l_{t+1}^1}{l_t^1}\right)^{\sigma_1+\theta_1-1}\left(\frac{Q_{t+1}}{Q_t}\right)^{\gamma_1} = 1.$$

By taking the logarithm of both sides, it can be rewritten in the following simpler form:

$$\Delta \ln Q_{t+1} + \rho_1^1 \Delta \ln l_{t+1}^1 = -\rho_2^1 \Delta \ln w_{1t+1} - \rho_3^1 \ln\left(\frac{p_t}{p_{t+1}}\right) - \rho_4^1 \ln\left(R_{t+1}\right) - \rho_4^1 \ln \beta. \tag{3}$$

where $\rho_1^1 = \frac{\sigma_1+\theta_1-1}{\gamma_1}$, $\rho_2^1 = \frac{\sigma_1-1}{\gamma_1}$, $\rho_3^1 = \frac{\sigma_1}{\gamma_1}$, and $\rho_4^1 = \frac{1}{\gamma_1}$. Two features of this optimality condition are worth a discussion. First, this equation depends on five unknown parameters ρ_1^1, ρ_2^1, ρ_3^1, ρ_4^1, and β, and observed variables. Second, in an environment with uncertainty one needs this equation plus four additional moment conditions to identify the five parameters. The standard approach in the estimation of parameters contained in Euler equations is to use lagged variables to construct the four additional moment conditions. With the goal of providing some insight on the variation required in the data to identify these parameters, instead of considering the standard case with uncertainty I will discuss the case of no uncertainty.

Consider a change in the amount of resources allocated to period t, $\bar{Y}_t$, generated for instance by a variation in the father's wage in period $t' \neq t,\ t+1$. The household will respond to this variation by changing how the father's leisure, the father's private consumption, and public consumption evolve between t and $t+1$. This intertemporal change depends on the father's taste for leisure and private consumption relative to his taste for public consumption, which is described by ρ_1^1. It can

be determined by differentiating the father's private consumption Euler equation (3) with respect to $\bar{Y}_t$ and it can be described using the following equation:

$$\Delta \ln Q_{\bar{Y}_t,t+1} + \rho_1^1 \Delta \ln l^1_{\bar{Y}_t,t+1} = 0,$$

where $Q_{\bar{Y}_t,t+1}$ and $l^i_{\bar{Y}_t,t+1}$ are the partial derivatives of public consumption and leisure with respect to $\bar{Y}_t$. This implies that one can recover the father's taste for leisure and private consumption relative to his taste for public consumption by simply observing the intertemporal change in father's leisure and public consumption in response to a change in resources available in a given period, i.e.

$$\rho_1^1 = -\frac{\Delta \ln Q_{\bar{Y}_t,t+1}}{\Delta \ln l^1_{\bar{Y}_t,t+1}}.$$

Now that the relative taste parameter ρ_1^1 is known, it is straightforward to identify the parameter ρ_3^1 which provides information on the father's taste for private consumption relative to his test for public consumption. Consider a change in the price of private consumption at t. The household varies the father's leisure and public consumption according to the following optimality condition:

$$\Delta \ln Q_{p_t,t+1} + \rho_1^1 \Delta \ln l^1_{p_t,t+1} = -\rho_3^1 \frac{1}{p_t}.$$

As a consequence, ρ_3^1 can be recovered if one observes a change in p_t and the corresponding change in leisure and public consumption. Specifically,

$$\rho_3^1 = -p_t \left(\Delta \ln Q_{p_t,t+1} + \rho_1^1 \Delta \ln l^1_{p_t,t+1} \right).$$

Finally one can recover the parameter ρ_2^1, which provides different information on the father's taste for private consumption relative to his taste for public consumption, if variation in the father's wage in period t is observed. The effect of this variation on the father's leisure and public consumption can be determined by differentiating the father's private consumption Euler equation with respect to his wage. The following equation describes the effect:

$$\Delta \ln Q_{w_{1t},t+1} + \rho_1^1 \Delta \ln l^1_{w_{1t},t+1} = \rho_2^1 \frac{1}{w_{1t}}.$$

The parameter ρ_2^1 is therefore equal to

$$\rho_2^1 = w_{1t} \left(\Delta \ln Q_{w_{1t},t+1} + \rho_1^1 \Delta \ln l^1_{w_{1t},t+1} \right).$$

Now that the reduced form parameters ρ_1^1, ρ_2^1, and ρ_3^1 are known, it is straightforward to recover the father's preference parameters for the non-separable part of his utility function. They are equal to the following functions of the reduced form parameters:

$$\sigma_1 = \frac{\rho_3^1}{\rho_3^1 - \rho_2^1}, \quad \theta_1 = \frac{\rho_1^1 - \rho_2^1}{\rho_3^1 - \rho_2^1}, \quad \gamma_1 = \frac{1}{\rho_3^1 - \rho_2^1}.$$

Since γ_1 has been recovered, the reduced-form parameter ρ_4^1 is also known. The discount factor can then be identified by solving the private consumption Euler equation for β.

The father's private consumption can also be recovered since it only depends on the parameters σ_1 and θ_1. As a consequence, the mother's private consumption is also identified. It should be remarked that individual consumption can be identified only because of the particular functional form chosen for the utility functions. In general, individual consumption can be identified only up to an additive constant. In the example considered here, the constant is assumed to be zero.

I will now describe how the mother's preference parameters for private consumption and the non-separable part of her preferences for public consumption can be recovered using her private consumption Euler equation. Her taste for leisure, however, cannot be identified since no variation in her labor supply is observed. Since the mother's private consumption is now known, there is no need to substitute out private consumption from her Euler equation, which can be written in the form

$$\Delta \ln Q_{t+1} + \rho_1^2 \Delta \ln c_{t+1}^2 = -\rho_2^2 \ln \left(R_{t+1} \frac{p_t}{p_{t+1}} \right) - \rho_2^2 \ln \beta,$$

where $\rho_1^2 = \frac{\sigma_2 - 1}{\gamma_2}$ and $\rho_2^2 = \frac{1}{\gamma_2}$. The identification of the mother's preference parameters can be achieved using the logic used for the father. Consider a change in the resources available in period t generated by a variation in one of the exogenous variables in period $t' \neq t,\ t+1$. This change modifies how the household allocates resources between t and $t+1$. The corresponding intertemporal change for the mother can be described by differentiating her private consumption Euler equation with respect to $\bar{Y}_t$, i.e.

$$\Delta \ln Q_{\bar{Y}_t, t+1} + \rho_1^2 \Delta \ln c_{\bar{Y}_t, t+1}^2 = 0.$$

This type of variation enables one to recover the mother's taste for private consumption relative to her taste for public consumption ρ_1^2, i.e.

$$\rho_1^2 = -\frac{\Delta \ln Q_{\bar{Y}_t, t+1}}{\Delta \ln c_{\bar{Y}_t, t+1}^2},$$

The inverse of the taste for public consumption ρ_2^2 can now be recovered if variation in the price of private consumption at t and the corresponding changes in intertemporal decisions are observed. These changes are described by the following equation:

$$\Delta \ln Q_{p_t,t+1} + \rho_1^2 \Delta \ln c^2_{p_t,t+1} = -\rho_2^2 \frac{1}{p_t}.$$

which implies that

$$\rho_2^2 = -p_t \left(\Delta \ln Q_{p_t,t+1} + \rho_1^2 \Delta \ln c^2_{p_t,t+1} \right).$$

Finally, the mother's preference parameters for private and public consumption can be recovered using the information on the reduced-form parameters ρ_1^2 and ρ_2^2. Specifically,

$$\sigma_2 = \frac{\rho_1^2 + \rho_2^2}{\rho_2^2}, \quad \gamma_2 = \frac{1}{\rho_2^2}.$$

Only three of the parameters of interest remain to be identified: the decision power parameter μ and the parameters that describe the separable part of the individual preferences for public consumption δ_1 and δ_2. Intuitively, one should expect that these parameters cannot be identified by simply using the private consumption Euler equations, since they provide no information on the individual decision power and on the separable part of the preferences for the public good. Some additional restrictions imposed by the model on individual behavior must be employed to identify the remaining parameters.

The decision power parameter can be recovered using the private consumption efficiency condition (2). In this equation all the parameters and variables are known except μ. One can therefore identify the individual decision power by solving this equation for μ. The parameters δ_1 and δ_2 can be recovered using the public consumption efficiency condition in two different periods. The public consumption efficiency condition at t and $t+1$ can be written in the following form:

$$A_t^1 + \delta_1 B_t^1 + A_t^2 + \delta_2 B_t^2 = \frac{P_t}{p_t},$$

and

$$A_{t+1}^1 + \delta_1 B_{t+1}^1 + A_{t+1}^2 + \delta_2 B_{t+1}^2 = \frac{P_{t+1}}{p_{t+1}}.$$

where A_t^i, B_t^i, A_{t+1}^i and B_{t+1}^i are functions of known parameters and observed variables. The parameters that characterize the separable part of the individual preferences for public consumption

can therefore be recovered by solving these two equations for δ_1 and δ_2. They can be written in the form

$$\delta_1 = \frac{B_t^2 A_{t+1}^1 + B_t^2 A_{t+1}^2 - A_t^1 B_{t+1}^2 - A_t^2 B_{t+1}^2 - B_t^2 P_{t+1} + P_t B_{t+1}^2}{B_t^1 B_{t+1}^2 - B_{t+1}^1 B_t^2},$$

and

$$\delta_2 = \frac{B_t^1 A_{t+1}^1 + B_t^1 A_{t+1}^2 - B_{t+1}^1 A_t^1 - B_{t+1}^1 A_t^2 + B_{t+1}^1 P_t - B_t^1 P_{t+1}}{B_{t+1}^1 B_t^2 - B_t^1 B_{t+1}^2}.$$

The results presented in this section suggest that the parents' preferences for expenditure on children can be identified even if only one parent works and the individual preferences are non-separable in public consumption. The information on preferences for expenditure on children can be used to predict how much parents in different income quartiles will invest in their children. Policies that attempt to improve the cognitive and non-cognitive abilities of children living in deprived environments can then be designed to reflect potential differences in early investments across income quartiles.

4 A General Identification Result

The identification result presented in the previous section will be extended to the general set of utility functions described in section 2. The result is a generalization to an environment with expenditure on children of the identification result obtained in Mazzocco (2005), where it is shown that intertemporal preferences can be recovered.

Identification is achieved in four steps. In the first step, individual consumption is derived as a function of observed variables using the optimality condition that relates the individual marginal rate of substitution between consumption and leisure to own real wage. In the second step, individual consumption is substituted out of the individual marginal utilities using the consumption function obtained in the first step. In the third step, intra-period and intertemporal optimality conditions are derived using the reduced-form marginal utilities obtained in the second step. It is then shown that the reduced-form marginal utilities and individual decision power can be identified using this set of conditions. In the last step, the individual utilities are recovered exploiting the information on the reduced-form marginal utility functions.

Suppose that in each period at least one agent chooses to supply a positive amount of labor. Without loss of generality, it will be assumed that agent 1 satisfies this restriction. Under this

assumption, the first order conditions at t for the intertemporal collective model imply that agent 1's marginal rate of substitution between private consumption and leisure must equal the real wage, i.e.,

$$\frac{u_l^1\left(c_t^1, 1-h_t^1, Q_t\right)}{u_c^1\left(c_t^1, 1-h_t^1, Q_t\right)} = q\left(c_t^1, h_t^1, Q_t\right) = \bar{w}_{1t},$$

where $\bar{w}_{1t} = \dfrac{w_{1t}}{p_t}$. If the inverse function of q is well-defined, agent 1's consumption can be written as the following unknown function of individual labor supply, public consumption, and real wage:[7]

$$c_t^1 = g\left(\bar{w}_{1t}, h_t^1, Q_t\right).$$

Since household private consumption is observed and in each period $C_t = c_t^1 + c_t^2$, agent 2's private consumption can also be written as a function of observed variables as follows:

$$c_t^2 = C_t - g\left(\bar{w}_{1t}, h_t^1, Q_t\right).$$

Using the function g, the unobserved individual private consumption can be substituted out of the marginal utilities that define the intratemporal and intertemporal optimality conditions. Denote with f_k^1 and f_k^2 the reduced-form marginal utilities with respect to good k for agent 1 and 2 obtained with this substitution. Then f_k^1 and f_k^2 can be defined as follows:

$$f_k^1\left(\bar{w}_{1t}, h_t^1, Q_t\right) = u_k^1\left(g\left(\bar{w}_{1t}, h_t^1, Q_t\right), 1-h_t^1, Q_t\right) \quad k = c, l, Q, \tag{5}$$

$$f_k^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right) = u_k^2\left(C_t - g\left(\bar{w}_{1t}, h_t^1, Q_t\right), 1-h_t^2, Q_t\right) \quad k = c, l, Q. \tag{6}$$

An example for f_k^1 and f_k^2 can be easily derived using the parametric specification assumed in the previous section. For instance, in that case the father's reduced-form marginal utility for private consumption is characterized by the reduced form parameters $\alpha_1 = \sigma_1 - 1$, $\alpha_2 = \sigma_1 + \theta_1 - 1$, $\alpha_3 = \sigma_1\left(\frac{\sigma_1}{\theta_1}\right)^{\alpha_1}$, and by the preference parameter γ_1.

The reduced-form marginal utilities can be used to rewrite the individual private consumption Euler equations in terms of observed variables. To that end, the assumption that agent 1 supplies

[7] The consumption function g is well-defined if the marginal rate of substitution q is strictly increasing in consumption, which is a standard assumption in the labor literature. More formally, lemma 1 in the appendix shows that g is well-defined if

$$u_{lc}^1\left(c_t^1, 1-h_t^1, Q_t\right) u_c^1\left(c_t^1, 1-h_t^1, Q_t\right) - u_{cc}^1\left(c_t^1, 1-h_t^1, Q_t\right) u_l^1\left(c_t^1, 1-h_t^1, Q_t\right) \neq 0. \tag{4}$$

The function g corresponds to the m-consumption function introduced by Browning (1998).

a positive amount of labor must be fulfilled for two consecutive periods. Under this restriction, the intertemporal optimality conditions can be written as follows:

$$f_c^1\left(\bar{w}_{1t}, h_t^1, Q_t\right) = \beta E_t\left[f_c^1\left(\bar{w}_{1t+1}, h_{t+1}^1, Q_t\right) R_{t+1}\frac{p_t}{p_{t+1}}\right],$$

$$f_c^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right) = \beta E_t\left[f_c^2\left(C_{t+1}, \bar{w}_{1t+1}, h_{t+1}^1, h_{t+1}^2, Q_t\right) R_{t+1}\frac{p_t}{p_{t+1}}\right].$$

Since household private consumption, public consumption, individual labor supply, individual wages, and the interest rate are observed, the reduced-form marginal utilities f_c^1 and f_c^2, and the discount factor β can be identified using the private consumption Euler equations and methods that have been developed for the identification of Euler equations.[8]

The remaining reduced-form marginal utilities can be identified using the intra-period optimality conditions and the public consumption Euler equation. Observe that agent 1's marginal rate of substitution between private consumption and leisure must be equal to the real wage even if individual consumption is substituted out using the consumption function g. This implies that

$$\frac{u_l^1\left(g\left(\bar{w}_{1t}, h_t^1, Q_t\right), 1 - h_t^1, Q_t\right)}{u_c^1\left(g\left(\bar{w}_{1t}, h_t^1, Q_t\right), 1 - h_t^1, Q_t\right)} = \frac{f_l^1\left(\bar{w}_{1t}, h_t^1, Q_t\right)}{f_c^1\left(\bar{w}_{1t}, h_t^1, Q_t\right)} = \bar{w}_{1t}.$$

Now consider a realization of the exogenous variables p_t, P_t, w_{1t}, w_{2t}, and of the amount of resources $\bar{Y}_t$, which are all observed.[9] Conditional on this realization, the household members choose the optimal amount of C_t, Q_t, h_t^1, and h_t^2, which are also observed. For the observed $\bar{w}_{1t}$, h_t^1, Q_t, the function $f_c^1\left(\bar{w}_{1t}, h_t^1, Q_t\right)$ is known from the private consumption Euler equations. Consequently, one can recover $f_l^1\left(\bar{w}_{1t}, h_t^1, Q_t\right)$ for the observed $\bar{w}_{1t}$, h_t^1, and Q_t by setting it equal to $\bar{w}_{1t} f_c^1\left(\bar{w}_{1t}, h_t^1, Q_t\right)$. By using the same argument for every realization of p_t, P_t, w_{1t}, w_{2t}, and $\bar{Y}_t$, the entire function f_l^1 can be identified.

The individual decision power can be identified using a similar idea. The private consumption efficiency condition can be written using the reduced-form marginal utilities in the following form:

$$\frac{f_c^1\left(\bar{w}_{1t}, h_t^1, Q_t\right)}{f_c^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right)} = \frac{1-\mu}{\mu}.$$

[8] As mentioned in the introduction, in principle the Euler equations can be identified non-parametrically. However, until a paper on non-parametric identification of Euler equations is written the identification result of this paper relies on parametric methods of identification of Euler equations.

[9] Note that $\bar{Y}_t$ is not exogenous but it depends on all exogenous variables in each period. It can therefore be varied by changing one of the exogenous variables at $t' \neq t$. In the remainder of the section, it will therefore be treated as an exogenous variable.

For every realization of p_t, P_t, w_{1t}, w_{2t}, and $\bar{Y}_t$, the functions f_c^1 and f_c^2 are known from the Euler equations. The relative decision power μ is therefore also identified.

The reduced-form marginal utilities of public consumption can be recovered using the public consumption Euler equation and the public consumption efficiency condition. To understand how these functions can be recovered, note that the public consumption Euler equation can be written in the form

$$f_Q^1\left(\bar{w}_{1t}, h_t^1, Q_t\right) + \frac{1-\mu}{\mu} f_Q^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right) =$$
$$\beta E_t\left[\left(f_Q^1\left(\bar{w}_{1t+1}, h_{t+1}^1, Q_{t+1}\right) + \frac{1-\mu}{\mu} f_Q^2\left(C_{t+1}, \bar{w}_{1t+1}, h_{t+1}^1, h_{t+1}^2, Q_{t+1}\right)\right) R_{t+1} \frac{P_t}{P_{t+1}}\right],$$

where μ and β are known from the private consumption efficiency condition and Euler equations. The public consumption Euler equation enables one to recover the following function:

$$G\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right) = f_Q^1\left(\bar{w}_{1t}, h_t^1, Q_t\right) + \frac{1-\mu}{\mu} f_Q^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right). \tag{7}$$

Household decisions must also satisfied the following public consumption efficiency condition:

$$\frac{f_Q^1\left(\bar{w}_{1t}, h_t^1, Q_t\right)}{f_c^1\left(\bar{w}_{1t}, h_t^1, Q_t\right)} + \frac{f_Q^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right)}{f_c^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right)} = \frac{P_t}{p_t}. \tag{8}$$

Note that for every realization of p_t, P_t, w_{1t}, w_{2t}, and $\bar{Y}_t$ the functions f_c^1 and f_c^2, and the decision power parameter μ are known. The reduced-form marginal utilities for public consumption can therefore be identified by solving equations (7) and (8) for f_Q^1 and f_Q^2 for every realization of p_t, P_t, w_{1t}, w_{2t}, and $\bar{Y}_t$.

Under the additional assumption that agent 2 chooses to supply a positive amount of labor, agent 2's reduced-form marginal utility of leisure can also be identified by equating her marginal rate of substitution between private consumption and leisure to the real wage, i.e.,

$$\frac{f_l^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right)}{f_c^2\left(C_t, \bar{w}_{1t}, h_t^1, h_t^2, Q_t\right)} = \bar{w}_{2t}.$$

The function f_c^2 is known from the Euler equations for every realization of the exogenous variables p_t, P_t, w_{1t}, w_{2t}, and $\bar{Y}_t$. Thus, f_l^2 can be identified by setting it equal to $\bar{w}_{2t} f_c^2$.

All the reduced-form marginal utilities are therefore identified. However, the information on individual preferences is contained in the original marginal utilities. The following proposition shows that the original marginal utilities are identified if the reduced-form marginal utilities are known and variation in all the exogenous variables is observed.

Proposition 1 *If both agents supply a positive amount of labor and either* u^1 *or* u^2 *satisfies the invertibility condition (4), the marginal utilities* u^1_c, u^2_c, u^1_l, u^2_l, u^1_Q, u^2_Q, *the decision power* μ, *and the consumption function* g *are identified up to the additive constant of* g.

If only agent 1 *supplies a positive amount of labor and* u^1 *satisfies the invertibility condition (4), all the marginal utilities are identified except the marginal utility of leisure for the spouse that does not work. Moreover,* μ *and* g *are identified up to the additive constant of* g.

Proof. In the appendix. ■

To provide the intuition underlying proposition 1, note that if the function $g\left(\bar{w}_1, h^1, Q\right)$ is known the original marginal utilities can be easily identified by means of equations (5) and (6). I will now discuss how $g\left(\bar{w}_1, h^1, Q\right)$ can be recovered using variation in variables that are observed in the data. Since some insight for the case of a household with only one worker was provided in the previous section, here I will consider the case of a household in which both parents work. Equation (6) implies that for every realization of the exogenous variables agent 2's reduced-form marginal utilities of private and public consumption must satisfy the following identities:

$$f^2_c\left(C, \bar{w}_1, h^1, h^2, Q\right) = u^2_c\left(C - g\left(\bar{w}_1, h^1, Q\right), 1 - h^2, Q\right). \tag{9}$$

and

$$f^2_Q\left(C, \bar{w}_1, h^1, h^2, Q\right) = u^2_Q\left(C - g\left(\bar{w}_1, h^1, Q\right), 1 - h^2, Q\right). \tag{10}$$

Consider variations in the exogenous variables that generate a group of households with identical $\bar{w}_1$, h_1, h_2, and Q but different C. This group of households enables one to recover $u^2_{c,c}$, i.e. how agent 2's marginal utility of private consumption varies with agent 2's private consumption holding everything else constant. To see this observe that f^2_c is known, which implies that it is known how f^2_c varies with C if $\bar{w}_1$, h_1, h_2, and Q are held constant. Since (9) is satisfied for every feasible C, how u^2_c varies with C holding $\bar{w}_1$, h_1, h_2, and Q constant must be equivalent to how f^2_c varies with C if $\bar{w}_1$, h_1, h_2, and Q are held constant. Finally, how u^2_c varies with C holding $\bar{w}_1$, h_1, h_2, and Q constant corresponds to u^2_{cc}. Consequently, $u^2_{cc} = f^2_{cc}$. The same argument applied to equation (10) implies that $u^2_{Qc} = f^2_{Qc}$.

Consider now changes in the exogenous variables that generate the group of households for which C, $\bar{w}_1$, h_2, and Q are constant, but h_1 varies. This group of households provides joint

information on u^2_{cc} and g_{h_1}. To explain this note that it is known how f^2_c varies with h_1 if C, $\bar{w}_1$, h_2, and Q are held constant. By (9), how u^2_c varies with h^1 holding C, $\bar{w}_1$, h_2, and Q constant must be equivalent to how f^2_c varies with h_1 if C, $\bar{w}_1$, h_2, and Q are held constant. Finally, observe that by varying h^1 on the right hand side of (9), one obtains information on $u^2_{cc}g_{h_1}$. This implies that $u^2_{cc}g_{h_1} = -f^2_{ch_1}$.

Consider the variation in the exogenous variables that generates the group of households for which C, h_1, h_2, and Q are constant, but $\bar{w}_1$ varies. Using the argument employed for the previous group of households, it can be shown that $u^2_{cc}g_{\bar{w}_1} = -f^2_{c\bar{w}_1}$. Consider the variation in p_t, P_t, w_{1t}, w_{2t}, and $\bar{Y}_t$ that generates the group of households for which C, h_1, h_2, and $\bar{w}_1$ are constant, but Q varies. The logic employed for the previous two groups of households indicates that $-u^2_{cc}g_Q + u^2_{cQ} = f^2_{c\bar{w}_1}$.

All the information required to identify how $g\left(\bar{w}^1, h^1, Q\right)$ varies with $\bar{w}_1$, h_1, and Q is now known. Using the first and second group of households one obtains that $g_{h_1} = -f^2_{ch_1}/f^2_{cc}$. The first and third group of households imply that $g_{\bar{w}_1} = -f^2_{c\bar{w}_1}/f^2_{cc}$. Using the first and fourth group of households it can be shown that $g_Q = \left(f^2_{Qc} - f^2_{c\bar{w}_1}\right)/f^2_{cc}$. Finally, since it is known how $g\left(\bar{w}^1, h^1, Q\right)$ varies with $\bar{w}_1$, h_1, and Q, the function g is known up to an additive constant. It is then straightforward to recover the original marginal utilities using the reduced-form marginal utilities and g.

An implication of Proposition 1 is that the individual preferences over private consumption, public consumption, and leisure can be identified. This leads to the following corollary.

Corollary 1 *If both parents work, the individual preferences over public consumption, private consumption, and leisure are identified up to an additive constant.*

If only one parent works, the individual preferences over public and private consumption for both parents and the preferences over leisure for the working parent are identified up to an additive constant.

This Corollary indicates that the preferences for expenditure on children of the mother and father can be identified even if only one of them supplies a positive amount of labor hours. This result should help researchers in predicting which of different policies designed to improve the cognitive and non-cognitive abilities of children will be the most effective.

The implementation of the identification method proposed in this paper requires a longitudinal

dataset that contains information on leisure, public consumption, private consumption, and wages. A dataset with these features is the CEX, which is a longitudinal dataset with information on all the required variables. Individual labor supply and wages are observed. Detailed data on expenditure on different consumption items are collected. Moreover, the expenditure data include information on the main components of children expenditure, namely expenditure on children clothing, children shoes, school books, and other educational expenses. All these variables are observed for four consecutive quarters. A drawback of the CEX is that food consumption is only measured at the household level. It is therefore not possible to determine which fraction is consumed by children, which represents public consumption. As a partial solution, the econometrician can either assume that food consumption is separable from other consumption goods and leisure or she can impute the fraction of food items consumed by children using information on the type of goods purchased by the household and the family structure.

5 Final Remarks

This paper shows that the parents' preferences for expenditure on children can be recovered without separability assumptions if at least one of the parents supplies a positive amount of labor. This result can be used to evaluate the performance of different policies designed to improve test scores and future earnings of children raised in deprived environments.

There is one weakness of the proposed approach that is worth a discussion. In the paper it is implicitly assumed that domestic labor is exogenously determined. Under this assumption, it can be incorporated in the time endowment. An important project which is left for future research is to generalize the identification result to an environment that allows for endogenous choices of domestic labor. In the meanwhile, as a partial solution empirical works should model the time endowment as a function of exogenous variables that determine domestic labor. In this way, differences across households in domestic labor are captured by the heterogeneity in time endowment.

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A Proofs

A.1 Lemma 1

The following Lemma determines the condition under which the marginal rate of substitution function q can be inverted and therefore the consumption function g is well-defined.

Lemma 1 *The function* $g\left(\bar{w}_{1t}, h_t^1, Q_t\right)$ *is well-defined if*

$$u_{lc}^1\left(c_t^1, 1-h_t^1, Q_t\right) u_c^1\left(c_t^1, 1-h_t^1, Q_t\right) - u_{cc}^1\left(c_t^1, 1-h_t^1, Q_t\right) u_l^1\left(c_t^1, 1-h_t^1, Q_t\right) \neq 0,$$

for any realization of the exogenous variables.

Proof. For any realization of the exogenous variables define

$$d^1\left(c^1, h^1, Q, \bar{w}_1\right) = q^1\left(c_t^1, h_t^1, Q\right) - \bar{w}_{1t} = 0.$$

By the implicit function theorem, $g^1\left(\bar{w}_1, h^1, Q\right)$ is well-defined if $\dfrac{\partial d^1}{\partial c^1} \neq 0$. Which implies the result. ■

A.2 Proof of Proposition 1

In the second stage of the household problem, the household chooses optimal consumption and leisure in each period and state of nature given $w_{1,t,\omega}$, $w_{2,t,\omega}$, $p_{t,\omega}$, $P_{t,\omega}$, and $\bar{Y}_{t,\omega}$ according to the following problem:

$$\max_{c_{t,\omega}^1, c_{t,\omega}^2, l_{t,\omega}^1, l_{t,\omega}^2, Q_{t,\omega}} \mu u^1(c_{t,\omega}^1, l_{t,\omega}^1, Q_{t,\omega}) + (1-\mu)\, u^2(c_{t,\omega}^2, l_{t,\omega}^2, Q_{t,\omega})$$

$$s.t. \sum_{i=1}^{2}\left(p_{t,\omega} c_{t,\omega}^i + w_{i,t,\omega} l_{t,\omega}^i\right) + P_{t,\omega} Q_{t,\omega} \leq \bar{Y}_{t,\omega}$$

The price of the private good, $p_{t,\omega}$, the price of the public good, $P_{t,\omega}$, agent 1's wage, $w_{1,t,\omega}$, and agent 2's wage, $w_{2,t,\omega}$, represent four independent sources of exogenous variation. The fifth source of variation is $\bar{Y}_{t,\omega}$. It is important to remark that $\bar{Y}_{t,\omega}$ is endogenously determined and it is a function of the exogenous variables in any period and state of nature. This has two implications. First, a change in one of the exogenous variables at $t' \neq t$ and $\omega' \neq \omega$ varies $\bar{Y}_{t,\omega}$. Second, a change in an exogenous variable at $t' \neq t$ and $\omega' \neq \omega$ can vary household decisions in period t and state ω

only through $\bar{Y}_{t,\omega}$. In the remainder of the proof a change in $\bar{Y}_{t,\omega}$ should be interpreted as a change in an exogenous variable in period t' and state ω' that varies $\bar{Y}_{t,\omega}$.

Consider first the case in which both agents work. Note that if the function $g\left(\bar{w}_{1t}, h_t^1, Q_t\right)$ can be identified, the original marginal utilities can also be identified by means of the reduced-form marginal utilities which are known. In the remainder of the proof it will be shown that $\frac{\partial g}{\partial \bar{w}_1}$, $\frac{\partial g}{\partial h^1}$, and $\frac{\partial g}{\partial Q}$ can be identified, which implies that $g\left(\bar{w}_1, h^1, Q\right)$ can be identified up to an additive constant.

Consider an arbitrary period t and state ω. Given w_1, w_2, p, P, and $\bar{Y}$, optimal household private consumption, public consumption, agent 1's labor supply, and agent 2's labor supply can be written in the following form:

$$C = C\left(w_1, w_2, p, P, \bar{Y}\right), Q = Q\left(w_1, w_2, p, P, \bar{Y}\right), h^1 = h^1\left(w_1, w_2, p, P, \bar{Y}\right), h^2 = h^2\left(w_1, w_2, p, P, \bar{Y}\right).$$

Agent 2's reduced-form marginal utilities of private consumption and public consumption are defined as follows:

$$f_c^2\left(C, \bar{w}_1, h^1, h^2, Q\right) = u_c^2\left(C - g\left(\bar{w}_1, h^1, Q\right), 1 - h^2, Q\right). \tag{11}$$

$$f_Q^2\left(C, \bar{w}_1, h^1, h^2, Q\right) = u_Q^2\left(C - g\left(\bar{w}_1, h^1, Q\right), 1 - h^2, Q\right). \tag{12}$$

By construction these equations are satisfied for any combination of w_1, w_2, p, P, and $\bar{Y}$. Consider an arbitrary w_1, w_2, p, P, and $\bar{Y}$. Let dw_1, dw_2, dp, dP, and $d\bar{Y}$ be a small change in the exogenous variables with the following properties: (i) $dw_1 = \frac{w_1}{p}dp$, which implies that $d\bar{w}_1 = 0$; (ii) dw_2, dp, dP, and $d\bar{Y}$ are the solution of the following linear system:

$$\begin{aligned}
&\frac{\partial C}{\partial w_2}dw_2 + \left(\frac{\partial C}{\partial w_1}\frac{w_1}{p} + \frac{\partial C}{\partial p}\right)dp + \frac{\partial C}{\partial P}dP + \frac{\partial C}{\partial \bar{Y}}d\bar{Y} = dC \neq 0,\\
&\frac{\partial Q}{\partial w_2}dw_2 + \left(\frac{\partial Q}{\partial w_1}\frac{w_1}{p} + \frac{\partial Q}{\partial p}\right)dp + \frac{\partial Q}{\partial P}dP + \frac{\partial Q}{\partial \bar{Y}}d\bar{Y} = dQ = 0,\\
&\frac{\partial h^1}{\partial w_2}dw_2 + \left(\frac{\partial h^1}{\partial w_1}\frac{w_1}{p} + \frac{\partial h^1}{\partial p}\right)dp + \frac{\partial h^1}{\partial P}dP + \frac{\partial h^1}{\partial \bar{Y}}d\bar{Y} = dh^1 = 0,\\
&\frac{\partial h^2}{\partial w_2}dw_2 + \left(\frac{\partial h^2}{\partial w_1}\frac{w_1}{p} + \frac{\partial h^2}{\partial p}\right)dp + \frac{\partial h^2}{\partial P}dP + \frac{\partial h^2}{\partial \bar{Y}}d\bar{Y} = dh^2 = 0,
\end{aligned}$$

i.e., the change varies household private consumption, but household public consumption, agent 1's labor supply, and agent 2's labor supply stay constant. The change in f_c^2 implied by dw_1, dw_2,

dp, dP, and $d\bar{Y}$ can be computed as follows:[10]

$$df_c^2 = \frac{\partial f_c^2}{\partial C}dC + \frac{\partial f_c^2}{\partial \bar{w}_1}d\bar{w}_1 + \frac{\partial f_c^2}{\partial h^1}dh^1 + \frac{\partial f_c^2}{\partial h^2}dh^2 + \frac{\partial f_c^2}{\partial Q}dQ = \frac{\partial f_c^2}{\partial C}dC.$$

Similarly, the change in u_c^2 implied by dw_1, dw_2, dp, dP, and $d\bar{Y}$ can be written in the following form:

$$du_c^2 = -\frac{\partial u_c^2}{\partial c^2}dC.$$

Since equation (11) is satisfied for any w_1, w_2, p, P, and $\bar{Y}$, the change in f_c^2 must equal the change in u_c^2. Consequently,

$$\frac{\partial f_c^2}{\partial C} = -\frac{\partial u_c^2}{\partial c^2}.$$

Since $\dfrac{\partial f_c^2}{\partial C}$ is known, $\dfrac{\partial u_c^2}{\partial c^2}$ is also known.

Consider a change dw_1, dw_2, dp, dP, and $d\bar{Y}$ with the following properties: (i) $dw_1 = \dfrac{w_1}{p}dp$, which implies that $d\bar{w}_1 = 0$; (ii) dw_2, dp, dP, and $d\bar{Y}$ are the solution of the following linear system:

$$\begin{aligned}
&\frac{\partial C}{\partial w_2}dw_2 + \left(\frac{\partial C}{\partial w_1}\frac{w_1}{p} + \frac{\partial C}{\partial p}\right)dp + \frac{\partial C}{\partial P}dP + \frac{\partial C}{\partial \bar{Y}}d\bar{Y} = dC = 0,\\
&\frac{\partial Q}{\partial w_2}dw_2 + \left(\frac{\partial Q}{\partial w_1}\frac{w_1}{p} + \frac{\partial Q}{\partial p}\right)dp + \frac{\partial Q}{\partial P}dP + \frac{\partial Q}{\partial \bar{Y}}d\bar{Y} = dQ = 0,\\
&\frac{\partial h^1}{\partial w_2}dw_2 + \left(\frac{\partial h^1}{\partial w_1}\frac{w_1}{p} + \frac{\partial h^1}{\partial p}\right)dp + \frac{\partial h^1}{\partial P}dP + \frac{\partial h^1}{\partial \bar{Y}}d\bar{Y} = dh^1 \neq 0,\\
&\frac{\partial h^2}{\partial w_2}dw_2 + \left(\frac{\partial h^2}{\partial w_1}\frac{w_1}{p} + \frac{\partial h^2}{\partial p}\right)dp + \frac{\partial h^2}{\partial P}dP + \frac{\partial h^2}{\partial \bar{Y}}d\bar{Y} = dh^2 = 0,
\end{aligned}$$

i.e., the change varies agent 1's labor supply, but household private consumption, public consumption, and agent 2's labor supply stay constant. According to equation (11), the implied change in f_c^2 must equal the implied change in u_c^2. Consequently, the following equation must be satisfied:[11]

$$\frac{\partial f_c^2}{\partial h^1} = -\frac{\partial u_c^2}{\partial c^2}\frac{\partial g}{\partial h^1}.$$

Since $\dfrac{\partial f_l^2}{\partial h^1}$ and $\dfrac{\partial u_c^2}{\partial c^2}$ are known, $\dfrac{\partial g}{\partial h^1}$ is identified.

Consider a change dw_1, dw_2, dp, dP, and $d\bar{Y}$ with the following properties: (i) $dw_1 \neq \dfrac{w_1}{p}dp$, which implies that $d\bar{w}_1 \neq 0$; (ii) dw_1, dw_2, dp, dP, and $d\bar{Y}$ are the solution of the following linear

[10] Alternatively, one could totally differentiate f_c^2 with respect to the exogenous variables w_1, w_2, p, P, and $\bar{Y}$ and then impose the constraints implied by the system of linear equations.

[11] The steps used to derive this equation are equivalent to the steps used to derive (A.2).

system:

$$\begin{aligned}
\frac{\partial C}{\partial w_1}dw_1 + \frac{\partial C}{\partial w_2}dw_2 + \frac{\partial C}{\partial p}dp + \frac{\partial C}{\partial P}dP + \frac{\partial C}{\partial \bar{Y}}d\bar{Y} = dC = 0,\\
\frac{\partial Q}{\partial w_1}dw_1 + \frac{\partial Q}{\partial w_2}dw_2 + \frac{\partial Q}{\partial p}dp + \frac{\partial Q}{\partial P}dP + \frac{\partial Q}{\partial \bar{Y}}d\bar{Y} = dQ = 0,\\
\frac{\partial h^1}{\partial w_1}dw_1 + \frac{\partial h^1}{\partial w_2}dw_2 + \frac{\partial h^1}{\partial p}dp + \frac{\partial h^1}{\partial P}dP + \frac{\partial h^1}{\partial \bar{Y}}d\bar{Y} = dh^1 = 0,\\
\frac{\partial h^2}{\partial w_1}dw_1 + \frac{\partial h^2}{\partial w_2}dw_2 + \frac{\partial h^2}{\partial p}dp + \frac{\partial h^2}{\partial P}dP + \frac{\partial h^2}{\partial \bar{Y}}d\bar{Y} = dh^2 = 0,
\end{aligned}$$

i.e., the change does not vary household private consumption, public consumption, agent 1's labor supply, and agent 2's labor supply. By equation (11), the implied change in f_c^2 must equal the implied change in u_c^2, which implies that the following equation must be satisfied:

$$\frac{\partial f_c^2}{\partial \bar{w}_1} = -\frac{\partial u_c^2}{\partial c^2}\frac{\partial g}{\partial \bar{w}_1}.$$

Since $\frac{\partial f_c^2}{\partial \bar{w}_1}$ and $\frac{\partial u_c^2}{\partial c^2}$ are known, $\frac{\partial g}{\partial \bar{w}_1}$ is identified.

Similarly since the implied change in f_c^2 and f_Q^2 must equal the implied change in, respectively, u_c^2 and u_Q^2, the following equations must be satisfied:

$$\frac{\partial f_c^2}{\partial \bar{w}_1} = -\frac{\partial u_c^2}{\partial c^2}\frac{\partial g}{\partial \bar{w}_1},$$

$$\frac{\partial f_Q^2}{\partial \bar{w}_1} = -\frac{\partial u_Q^2}{\partial c^2}\frac{\partial g}{\partial \bar{w}_1}.$$

Note that $\frac{\partial f_c^2}{\partial \bar{w}_1}$, $\frac{\partial f_Q^2}{\partial \bar{w}_1}$, and $\frac{\partial g}{\partial \bar{w}_1}$ are known, which implies that $\frac{\partial u_Q^2}{\partial c^2}$ and $\frac{\partial u_c^2}{\partial c^2}$ are identified.

Finally, consider a change dw_1, dw_2, dp, dP, and $d\bar{Y}$ with the following properties: (i) $dw_1 = \frac{w_1}{p}dp$, which implies that $d\bar{w}_1 = 0$; (ii) dw_2, dp, dP, and $d\bar{Y}$ are the solution of the following linear system:

$$\begin{aligned}
\frac{\partial C}{\partial w_2}dw_2 + \left(\frac{\partial C}{\partial w_1}\frac{w_1}{p} + \frac{\partial C}{\partial p}\right)dp + \frac{\partial C}{\partial P}dP + \frac{\partial C}{\partial \bar{Y}}d\bar{Y} = dC = 0,\\
\frac{\partial Q}{\partial w_2}dw_2 + \left(\frac{\partial Q}{\partial w_1}\frac{w_1}{p} + \frac{\partial Q}{\partial p}\right)dp + \frac{\partial Q}{\partial P}dP + \frac{\partial Q}{\partial \bar{Y}}d\bar{Y} = dQ \neq 0,\\
\frac{\partial h^1}{\partial w_2}dw_2 + \left(\frac{\partial h^1}{\partial w_1}\frac{w_1}{p} + \frac{\partial h^1}{\partial p}\right)dp + \frac{\partial h^1}{\partial P}dP + \frac{\partial h^1}{\partial \bar{Y}}d\bar{Y} = dh^1 = 0,\\
\frac{\partial h^2}{\partial w_2}dw_2 + \left(\frac{\partial h^2}{\partial w_1}\frac{w_1}{p} + \frac{\partial h^2}{\partial p}\right)dp + \frac{\partial h^2}{\partial P}dP + \frac{\partial h^2}{\partial \bar{Y}}d\bar{Y} = dh^2 = 0,
\end{aligned}$$

i.e., the change varies public consumption, but it does not vary household private consumption, agent 1's labor supply, and agent 2's labor supply. According to (11), the implied change in f_c^2 must equal the implied change in u_c^2, which implies that

$$\frac{\partial f_c^2}{\partial Q} = -\frac{\partial u_c^2}{\partial c^2}\frac{\partial g}{\partial Q} + \frac{\partial u_c^2}{\partial Q}.$$

Observe that $\frac{\partial f_c^2}{\partial Q}$, $\frac{\partial u_Q^2}{\partial c^2}$, and $\frac{\partial u_c^2}{\partial c^2}$ are known. Consequently, $\frac{\partial g}{\partial Q}$ is identified.

Since $\frac{\partial g}{\partial h^1}$, $\frac{\partial g}{\partial \bar{w}_1}$, and $\frac{\partial g}{\partial Q}$ are known, the function g is identified up to the constant of integration. It is then straightforward to use $g\left(\bar{w}_1, h^1, Q\right)$ to recover u_c^i, u_l^i, and u_Q^i from f_c^i, f_l^i, and f_Q^i up to the additive constant of g.

It is important to remark that the proof requires that the following matrix of coefficients of the linear systems is of full rank:

$$\begin{bmatrix} \frac{\partial C}{\partial w_2} & \frac{\partial C}{\partial p} & \frac{\partial C}{\partial P} & \frac{\partial C}{\partial \bar{Y}} \\ \frac{\partial Q}{\partial w_2} & \frac{\partial Q}{\partial p} & \frac{\partial Q}{\partial P} & \frac{\partial Q}{\partial \bar{Y}} \\ \frac{\partial h^1}{\partial w_2} & \frac{\partial h^1}{\partial p} & \frac{\partial h^1}{\partial P} & \frac{\partial h^1}{\partial \bar{Y}} \\ \frac{\partial h^2}{\partial w_2} & \frac{\partial h^2}{\partial p} & \frac{\partial h^2}{\partial P} & \frac{\partial h^2}{\partial \bar{Y}} \end{bmatrix}.$$

There are two cases in which this condition is not satisfied: (i) at least one of the demand functions is independent of all the exogenous variables; (ii) the rows or columns are linearly dependent. Since the first case is not realistic, I will only discuss the second one. The rows of the matrix are linearly dependent if the variation in one of the demand functions generated by changes in the exogenous variables provides no additional information conditional on the variation in the other demand functions. The columns are linearly dependent if a change in one of the exogenous variables provide no additional information on how the demand functions C, Q, h^1, and h^2 vary conditional on the variation generated by the other exogenous variables. This emphasizes that the identification of individual preferences requires that independent variations in C, Q, h^1, and h^2 are observed and that the exogenous variables can generate it.

Consider the case in which only agent 1 supplies a positive amount of labor. In this case, h^2 is always equal to zero, no variation in w_2 is observed, and the reduced-form marginal utility f_l^2 is not known. In the first part of the proof, the equation defining f_l^2 and variation in h^2 were never used. Consequently, the previous argument can also be applied to households in which only

one agent supplies a positive amount of labor by dropping h^2 from equations (11) and (12), the corresponding linear equation from the three linear systems, and by setting $dw_2 = 0$. $g\left(\bar{w}_1, h^1, Q\right)$ is therefore identified up to the additive constant and all marginal utilities are identified except u_l^2.