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Strategic Information Disclosure: The Case of Multi-Attribute Products with Heterogeneous Consumers

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The Case of Multi-Attribute Products with Heterogeneous Consumers*

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Abstract

We examine the incentives for firms to voluntarily disclose otherwise private information about quality attributes of differentiated products. In particular, we focus on the case of differentiated products with multiple attributes and consumers that are heterogeneous in their preferences over these attributes. We show that there exist certain configurations of consumer preferences under which a firm producing a high-quality product, even with zero costs of disclosure, may choose not to reveal the quality of its product. This failure of firms to voluntarily disclose the quality of their products will arise when providing consumers with more information results in more elastic demands for these products, which, in turn, triggers more intensive price competition and leads to lower prices and profits for all firms. As a result, the equilibrium in which disclosure is voluntary may diverge from that in which disclosure is mandatory.

JEL: L15 (Information and Product Quality); L5 (Regulation and Industrial Policy)
Keywords**:** Quality Disclosure; Multiple Attributes; Consumer Heterogeneity

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1. Introduction

As technological innovation constantly expands the dimensionality of the product attribute space, the lack of information that consumers have about product quality has become of increasing concern for modern economies. However, one strand of economic literature, initiated independently by Grossman (1981) and Milgrom (1981), suggests that this concern is misplaced. In particular, Grossman and Milgrom argue that strong incentives exist in markets for firms to voluntarily disclose the quality of their products as long as there exists a verifiable disclosure mechanism with negligible costs. In equilibrium, firms' private information about the quality of their products would "unravel" and, as a result, a mandatory disclosure requirement would be redundant.[1] Furthermore, it follows that there would be little scope for firms to act strategically with respect to their disclosure decisions. In essence, high-quality firms, by distinguishing themselves from their lower-quality rivals, would always gain from revealing the true quality of their products, allowing them to charge consumers higher prices that reflect differences in quality. Only those with the lowest quality would choose not to reveal the quality of their products. However, their non-disclosure ends up being completely revealing because consumers correctly infer that a firm's failure to disclosure is always associated with the lowest-quality products.

If the Grossman and Milgrom conclusions about firms' product quality disclosure decisions hold, there are several phenomena that are somewhat difficult to explain. The first concerns the finding in a number of studies that consumer decisions do respond to the imposition of mandatory requirements that firms disclose the quality of their products. For example, Jin and Leslie (2003) find that sales of restaurants became more sensitive to restaurants' hygiene quality in Los Angeles County in the late 1990s after restaurants were required to post "grade cards" about their hygiene quality. They also find that as a result of these grade cards the incidence of hospitalizations due to food-borne illnesses declined in this county.

[1] The logic of their result is as follows. If no firm discloses its product quality, consumers will believe the highest-quality product is no different than the lower-quality ones. Firms producing products with the highest quality will want to disclose their quality, because they will then be able to charge a higher price and achieve higher profits. Firms with the next highest quality product have the same incentive to distinguish themselves from the remaining firms. This process continues, so long as the benefits of disclosure outweigh the costs. In the limit when disclosure

Similarly, Mathios (2000) finds that salad dressings with high fat content experienced large reductions in sales after the Nutritional Labeling and Education Act mandated all food products to carry standardized labels with information on a product's nutritional content.

Second, there is evidence that firms in some industries actively oppose the imposition of laws and regulations that require mandatory disclosure. For example, lobbying by the National Automobile Dealers Association in 1976 led to the abolishment of the Federal Trade Commission's mandatory inspection and disclosure rules on used-car dealers. In 1998, the National Restaurant Association strongly opposed the imposition of a requirement that all restaurants in Los Angeles County be required to publicly display "grade cards" that revealed each restaurant's hygiene practices.[2] Finally, the National Hospital Association opposed a proposal in 2000 by the Clinton administration to impose mandatory reporting on hospitals of all fatal and other serious medical errors.[3] If, as suggested by the results in Grossman and Milgrom, it is in the self-interest of firms—at least those with higher quality products—to reveal the quality of their products voluntarily, why would they, or their industry representatives, be willing to expend resources to avoid mandatory disclosure requirements?

In this paper, we examine a new model of firms' product quality disclosure decisions to assess the robustness of the Grossman (1981) and Milgrom (1981) unraveling result. Consistent with previous models, we consider the decisions of firms to voluntarily provide information about the quality of their products that would otherwise be unobserved by consumers.[4] We assume that firms have access to a mechanism with which they can credibly disclose such information at zero cost. There are two features that distinguish our model from the previous work in this literature. First, we allow for products that have multi-

costs are zero, the situation in which firms withhold private information about product quality "unravels."

[2] See *Food Council News*, Vol. 5, Issue 1, January 2002. The National Restaurant Association stated that the "rating initiatives reduce complex issues to a score or letter based on subjective decisions by individual inspectors." The industry maintained that if an establishment is good enough to pass a hygiene inspection, having to post ratings would constitute overkill.

[3] See *CNN News*, February 22, 2000. "A culture of silence" in the medical profession can be traced back to 1930s, when physicians were advised to "keep a cautious tongue" regarding medical errors (Gallagher et al., 2002).

[4] The literature we follow should be distinguished from the literature on information sharing among oligopolists as in Gal-Or (1985, 1986). In this paper, the product quality of each firm is known to all firms but unknown to con-

ple attributes, of which one is quality. Second, we assume that consumers have heterogeneous preferences over these attributes. Following the literatures on product differentiation and hedonic pricing, we consider a model in which products are both horizontally and vertically differentiated,[5] where quality is the vertical attribute. We show that a firm producing a high-quality product may actually benefit, rather than lose, from consumers not knowing about the quality of its product. As we establish below, this situation will arise if the demand for a differentiated product becomes more, rather than less, elastic with the disclosure of product quality. Whether consumer demand becomes more or less elastic after disclosure in turn depends on the distribution of consumer heterogeneity over *both* the horizontal and vertical attributes of products. If demand does becomes more elastic, disclosure of product quality will lead to more intense price competition between rival firms, causing prices and profits to fall for all firms. As such, a firm producing a high-quality product will find it in its interest not to disclose the quality of its product.

We note that there is a substantial literature that considers variants of the Grossman and Milgrom models of the product quality disclosure decision. Okuno-Fujiwara et al. (1990) derive sufficient conditions for complete revelation of all private information in equilibrium in a fairly general model. Researchers have also investigated certain types of disclosure costs (Jovanovic, 1982; Verrecchia, 1983; Dye, 1986), costs of information acquisition by sellers (Matthews and Postlewaite, 1985; Farrell, 1986; Shavell, 1994), consumers' limited understanding of sellers' disclosure (Fishman and Hagerty, 2003), consumers' uncertainty of the existence of information (Dye and Sridhar 1995; Stivers, 2004), and alternative market structures (Cheong and Kim, 2004; Board, 2005; Levin, Peck, and Ye, 2005). In these extensions, the failure to obtain voluntary disclosure from sellers, if it occurs, hinges on some form of "costs" associated with disclosure.[6] The basic conclusion that complete voluntary disclosure by sellers will occur in

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sumers.

[5] The work of Becker (1965), Lancaster (1966), Muth (1966), Rosen (1974), and Gorman (1980) views all goods and services as bundles of characteristics. Tastes are plausibly heterogeneous over these attributes and, thus, over the bundles. These two features—products viewed as bundles of characteristics and consumer heterogeneity—are the key elements of the literature on product differentiation and hedonic pricing.

[6] A notable exception is Board (2005), who considers a duopoly model with an outside option in which the lower-quality firm's profit function is non-monotonic in its perceived quality. In Board's model, a firm's (voluntary) non-disclosure decision hinges on the balancing of the incentives to alleviate competition with its higher-quality rivals

equilibrium continues to hold as long as these disclosure costs are zero. As has been noted elsewhere,[7] these models all lead to the rather strong conclusion that regulatory policies that mandate product quality disclosure are, at best, redundant when the costs of voluntary disclosure are zero. More precisely, this literature suggests that the only government intervention that would benefit consumers would be to ensure the availability of low-cost and credible mechanisms for sellers to disclose the quality of their products. As we establish below, high-quality firms may choose not to disclose their quality in equilibrium, even when disclosure costs are zero.

Our result concerning the possibility that the Grossman-Milgrom unraveling result may fail to hold is closely related to firms' decisions about whether to differentiate their products from those of rivals. The literature on product differentiation notes the inherent tension between the benefits to a firm from differentiating its products from those of its competitors and the desire to produce a product that is attractive to a large consumer base.[8] We show that the same tension arises in the product quality disclosure decisions of firms. Furthermore, while we assume that the attributes of products are exogenously given, we show that whether or not firms disclose the quality of their product to consumers represents a particular way in which firms can differentiate their products along some dimension.

The results of this paper are also related to the literature on informative advertising.[9] For example, Grossman and Shapiro (1984) consider the strategic effect of firms using advertising to provide truthful information about their location. They note that firms in an oligopoly setting may actually benefit from higher advertising costs and may even seek to raise such costs themselves. Higher advertising costs reduce the amount of advertising done in a market and leave consumers with less information to act upon

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with the firm's desire to improve consumer expectations about the quality of its product.

[7] Fishman and Hagerty (1997) offer a survey of this literature and discuss its policy implications.

[8] See Tirole (1994, Chapter 7) for an overview of the product differentiation literature. Most of this literature considers firms' product differentiation decisions for products that are horizontally and vertically differentiated separately. See Hotelling (1929), d'Aspremont, Gabszewicz, and Thisse (1979), and Neven (1985) for the former case and Mussa and Rosen (1978) and Gabszewicz and Thisse (1979 for the latter case. In a paper most closely related to the model we consider, Neven and Thisse (1990) examine firms' differentiation decisions for multi-attribute products that have both horizontal and vertical dimensions.

[9] Informative advertising refers to truthful provision of information about products' prices, attributes, etc.

when making their product choices. Less informed consumers will be less sensitive to prices across (differentiated) products and, as a result, firms can raise prices and achieve higher profits. It has sometimes been argued that this benefit of making advertising more costly is what has motivated professional organizations, such as the American Medical Association or the American Bar Association, to seek limitations on the amount of, if any, advertising done by its membership.[10] This incentive for strategic action is similar to the one that we argue below may discourage firms from voluntarily disclosing the quality of their products.

The remainder of the paper is organized as follows. In section 2, we develop duopoly model in which firms have products with two attributes, one readily observed by consumers and the other and consumers are assumed to have heterogeneous tastes over the two product attributes. In section 3, we provide an intuitive explanation of how disclosure can affect demand elasticities and in turn change firms' willingness to voluntarily disclose information. In section 4, we discuss some implications of our finding and offer some concluding remarks.

2. Model

2.1 Set Up

We consider a duopoly model in which two firms sell products with two attributes. Each firm, firm A and firm B, is endowed with a product Y_j, where $j = A, B$. These products differ across firms in their horizontal attribute, location (denoted by L),[11] and in their vertical attribute, the quality of their product (denoted by Q), so that $Y_j = Y(L_j,Q_j)$. We assume that these attributes are exogenously given and cannot be altered and that firms produce their products with the same production costs.[12] With respect to the horizontal attribute, firms are located vis-à-vis consumers at the end points of a Linear City, displayed in Figure 1. Let L_j

[10] See Peters (1984).

[11] Herein, location represents the horizontal attribute of a product. Horizontal attributes are those for which consumers have different preferences at the same prices, e.g., the color of a Honda Accord.

[12] We make this assumption to argue consumer beliefs do not vary with prices: prices do not depend on the value of any undisclosed quality and profit maximizing prices have no signaling roles.

denote the location of firm j in the linear city. Firm A's location is fixed at $L_A = 0$ and firm B's at $L_B = 1$. With respect to the vertical attribute, either firm can produce a high-quality product (q_h), or a low-quality one (q_l), but not both. We denote the difference between these two types of product qualities by Δ ($\Delta = q_h - q_l > 0$). Firm A is assumed to sell q_h with probability a_A and q_l with probability $1-a_A$, while firm B sells q_h with probability a_B and q_l with probability $1-a_B$. Finally, we assume that $0 \le a_A \le a_B \le 1$; that is, firm B is more likely to produce the high-quality product than firm A.

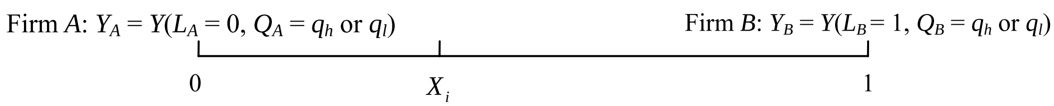

Figure 1: The Linear City

Consumers are uniformly distributed along the Linear City in Figure 1, where X_i ($0 \le X_i \le 1$) denotes the i^{th} consumer's location; thus, consumers differ with respect to their proximity to the two firms. We denote the distance of the i^{th} consumer from the j^{th} product as $D_{ij} = |L_j - X_j|$. In making their purchase decisions, consumers evaluate their transportation costs, their perceived product quality $E(Q_j)$, and the prices of products P_j. More precisely, the i^{th} consumer values product j according to the following utility function:

$$U_{ij}\left(D_{ij}, E(Q_j), P_j\right) \equiv V + \theta_i E(Q_j) - \lambda D_{ij} - P_j \tag{1}$$

where V is the stand-alone value of consuming either product (instead of consuming neither). We normalize the utility associated with consuming neither product to 0 and assume that V is high enough so no consumer will choose this outside option. We also have normalized the disutility associated with paying price P_j to 1. As specified in (1), consumers obtain the same disutility per unit of distance they must travel to purchase a particular product ($\lambda > 0$) but they differ with respect to the distances they have to travel. Basically λ measures the spread of consumer heterogeneity over the horizontal attribute.[13] To capture consumer heterogeneity with respect to the vertical attribute in a parsimonious way, we assume that there are two types of consumers. One type of consumers, "quality-lovers," value quality highly ($\theta_i = \theta$, where $\theta >$ 0) and the other type, "quality-neutrals," value it less ($\theta_i = \theta_0$, where $0 \leq \theta_0 \leq \theta$). We normalize θ_0 to 0; hence, the difference in the preference for quality across the two types of consumers is given by θ, since $\theta - \theta_0 = \theta - 0 = \theta$.

Consumer heterogeneity can be characterized by the joint distribution of consumers' locations and tastes for product quality. At the extreme, the distribution of consumers' locations and tastes for quality may be uncorrelated, i.e., $\Pr(X_i,\theta_i) = \Pr(X_i)\Pr(\theta_i)$. Alternatively, the distribution of consumer locations and tastes for quality may be correlated. To characterize this joint distribution in a tractable way, we assume that consumers' preferences for quality, conditional on their location are distributed as follows:

$$\Pr\left(\theta_i = \theta \middle| X_i\right) = \beta X_i + \frac{1-\beta}{2} \tag{2}$$

$$\Pr\left(\theta_i = 0 \middle| X_i\right) = -\beta X_i + \frac{1+\beta}{2} \tag{3}$$

where we assume that $\beta \in [0,1]$. It follows from (3) that the degree of correlation in consumer preferences over the horizontal and vertical attributes is characterized by the parameter, β. Consider the following two extreme cases:

Case I: $\beta = 0 \Rightarrow \Pr\left(\theta_i = \theta \middle| X_i\right) = \frac{1}{2}$ and $\Pr\left(\theta_i = 0 \middle| X_i\right) = \frac{1}{2}$

Case II: $\beta = 1 \Rightarrow \Pr\left(\theta_i = \theta \middle| X_i\right) = X_i$ and $\Pr\left(\theta_i = 0 \middle| X_i\right) = 1 - X_i$

In Case I, consumers' locations and tastes for quality are uncorrelated; quality-lovers and quality-neutrals are equally likely to be located at any point along the Linear City. In Case II, however, quality-lovers are more likely to live close to firm *B* (located at $L_B = 1$), and, thus, more likely to be closer to a high-quality product, q_h, while quality-neutrals are more likely to live close to firm *A* (located at $L_A = 0$), and, thus, more likely to be closer to a low-quality product, q_l. Case II, for example, might characterize the situation where firm *B* is located in the rich suburb of a city, where consumers are wealthier and have a greater ap-

[13] The greater the value of λ, the harder it is for consumers to travel from one end of the city to the other.

preciation for quality than the relatively poor residents who reside in the inner city, close to firm A.[14] As β increases from 0 to 1, the tendency of quality-lovers living closer to the high-end product gets larger and the two groups of consumers become further segregated. In short, β measures the degree of consumer segregation along the vertical attribute.[15]

To complete the model, we need to characterize what consumers and firms know and do not know about the product attributes of firms and the distribution of consumer locations and tastes for quality. We assume all consumers know their own preferences over quality, their own locations, and the locations of firms. Furthermore, consumers know the values of the parameters V, θ, Δ, λ, and β. As a result, consumers know how consumers are distributed with respect to their tastes for quality, i.e., they know the conditional probabilities in (2) and (3). Consumers are assumed *not* to know, a priori, the quality of either firm's product, at least not without each firm disclosing it. However, they do know the distribution of quality levels in the market, a_A and a_B.[16] With this information, consumers can form expectation about undisclosed quality, namely, $E(Q_A) = a_A q_h + (1-a_A)q_l$ and $E(Q_B) = a_B q_h + (1-a_B)q_l$. If firms provide consumers with any additional information about product quality, consumers will update their beliefs using Bayes' rule. Finally, we assume that firms, besides having all of the information consumers possess, know the quality of their own products and that of their rivals. In our model, firms possess private infor-

[14] This segregation can be a result of buyer-seller sorting, something which is not considered in this paper.

[15] Logically, we could allow $\beta \in [-1,1]$, i.e., consumers' locations and tastes for quality could be *positively* ($0 \leq \beta \leq 1$) or *negatively* correlated ($-1 \leq \beta \leq 0$). This would give rise to a third case, Case III: $\beta = -1 \Rightarrow \Pr(\theta_i = \theta \mid X_i) = 1-X$ and $\Pr((\theta_i = \theta \mid X_i) = X$. In Case III, quality-lovers would be more likely to live close to firm A and, thus, more likely to be closer to a low-quality product, q_l, while quality-neutrals would be more likely to live close to firm B, and, thus, more likely to be closer to a high-quality product, q_h. In a previous version of this paper, we allowed for the possibility of negatively correlated preferences. Moreover, allowing for a negative correlation of consumer locations and tastes for quality will tend to result in equilibria with non-disclosure by one or both firms. But, as noted by a referee, if consumers' locations were negatively correlated with their tastes for qualities, there would be a strong incentive for one (or both) of the firms to try to alter their products or locations. For example, firm A would have a clear incentive to produce a high-quality product in order to attract those quality-loving consumers residing near it. Then this firm could charge a higher price and realize a higher profit. As such, one would expect that in real world situations—where firms had some control over the quality of their products—such negative correlations would not exist in the long run. So as to make clear that our results on equilibria with less than complete disclosure about product quality is more than a theoretical curiosity, we rule out the possibility of this negative correlation case.

[16] That is, consumers know that four possible combinations of quality across the two firms can be offered, i..e., ($Q_A = q_h$, $Q_B = q_l$), ($Q_A = q_l$, $Q_B = q_l$), ($Q_A = q_l$, $Q_B = q_h$), and ($Q_A = q_h$, $Q_B = q_h$), and, given their knowledge of the probabilities, a_A and a_B, they know the probabilities with which these combinations can occur.

mation while consumers do not.

2.2 Disclosure Technology and Game Structure

We assume that there exists a truthful and costless disclosure mechanism for sellers to disclose high-quality products. In particular, we assume the existence of an agency that will certify that firms are selling a high-quality product at zero cost to the seller or consumers.[17] For example, the U.S. Food and Drug Administration (FDA) provides such certification, free of charge, for products ranging from bottled water to manufacturers of clinical diagnostic products that measure total cholesterol,[18] and *Good Housekeeping* magazine will provide and publicize its Good Housekeeping Seal of Approval for products it deems to be of high quality.[19] In our duopoly model, either firm that is endowed with a high-quality product can use this mechanism to credibly convey to consumers that it is selling a high-quality product. However, firms with low-quality products cannot use this mechanism and, thus, cannot directly reveal to consumers the true quality of its product. In the absence of a firm disclosing that it is selling a high-quality product, consumers must rely on their interpretation and inferences about the product's quality. If the "unraveling result" holds true, high-quality firms will always choose disclosure and low-quality firms will not. In this case, consumers correctly infer that non-disclosure is always associated with low quality and information is thus complete.[20]

To characterize the timing of firm decisions, we adopt a two-stage dynamic game. In the first stage, firms decide on disclosure simultaneously. In the second stage, firms engage in Bertrand competition to maximize profits and consumers choose which product to purchase so as to maximize their utility.

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[17] Lizzeri (1999) discusses the strategic manipulation of information by certification intermediaries and shows that a monopoly certification intermediary does not have full incentives to reveal all information. We assume away this strategic effect and focus on firms' incentives to use a truthful and non-strategic certification mechanism.

[18] See the FDA's website, www.fda.gov, for more on this governmental agency's certification functions.

[19] See this magazine's website, www.goodhousekeepingseal.com, for more on its product certification process.

[20] This assumption about disclosure technology is more restrictive than the one used in Grossman (1981) and Milgrom (1981), where firms at any quality level can disclose truthful and accurate information about their qualities. We introduce this restriction to avoid a low-quality firm's incentive to reveal its quality to be differentiated from its high-quality rival. The incentive of "revealing from the bottom" will complicate our model unnecessarily and redirect our study to product differentiation rather than information disclosure. Furthermore, most disclosure mechanisms in the real world will only allow qualities above a certain threshold to be revealed in a discrete manner, i.e.,

Consumers' valuations of products are conditional on product prices and what they have inferred about the quality of these products from what firms have disclosed about their products.

The essence of this two-stage game is that each firm makes its first-stage disclosure decisions, considering the resulting consumers' beliefs about product qualities and price competition between firms in the second stage. We solve for a subgame-perfect Bayesian Nash equilibrium, which consists of firms' equilibrium disclosure and pricing strategies, and consumers' equilibrium purchasing decisions and beliefs about product qualities. Specifically, in the first stage each firm evaluates its payoff (profit) in the second stage under various information scenarios, which are determined by its own disclosure decisions and that of its rival. Then the firm chooses the optimal first-stage strategy that yields the best final payoff, given consumers' beliefs and its rival's first-stage and second-stage responses taken into consideration. Consumers update their beliefs about firms' qualities using Bayes' rule after observing firms' disclosure decisions and then make purchasing decisions.

In section 2.3, we solve the second stage game given firms' disclosure decisions and consumers' beliefs. In section 2.4, we establish that firm *B*—the firm that has a higher probability of selling a high-quality product—has a dominant equilibrium disclosure strategy. However, firm *A*'s equilibrium disclosure strategy critically hinges on the nature of consumer heterogeneity, i.e., on the values of β, λ, θ, and Δ. In section 2.5, we solve the model and specify the Perfect Bayesian Equilibrium.

2.3 The Second-stage Outcomes Given Consumers' Beliefs

We solve the second stage outcomes given firms' first stage disclosure decisions and consumers' beliefs about qualities of the two firms' products. Recall that $\Delta = q_h - q_l$, i.e., Δ denotes the *real* difference between the high-quality and low-quality products. We further define $\tilde{\Delta} \equiv E(Q_B) - E(Q_A)$ as consumers *perceived* difference in quality between the two products, Y_B and Y_A, where the range for $\tilde{\Delta}$: $\tilde{\Delta} \in [-\Delta,\Delta]$ since either firm can produce a high-quality or low-quality product with positive probability.

We first consider quality-neutrals and quality-lovers separately:

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whether a product meets a certification agency's certification criteria.

- If a consumer is a quality-neutral ($\theta_i = 0$), she will purchase firm A's product $Y_A = Y(0,Q_A)$ if and only if $X_i \leq \dfrac{P_B - P_A + \lambda}{2\lambda}$ and firm B's product $Y_B = Y(1,Q_B)$ if and only if $X_i > \dfrac{P_B - P_A + \lambda}{2\lambda}$.

- If a consumer is a quality-lover ($\theta_i = \theta > 0$), she will purchase Y_A if and only if $X_i \leq \dfrac{P_B - P_A + \lambda - \theta\tilde{\Delta}}{2\lambda}$ and Y_B if and only if $X_i > \dfrac{P_B - P_A + \lambda - \theta\tilde{\Delta}}{2\lambda}$.

The purchase decisions of two different types of consumers are illustrated in Figure 2. In Figure 2, the horizontal axis represents the location of consumers, while the vertical axis represents the probability that a quality-lover lives at location X. The positively-sloped line in the figure divides the (X, Pr(θ)) space into two areas: area $C_1 + C_4$, which characterizes the "purchase regions" for quality-lovers, and area $C_2 + C_3$, which characterizes the corresponding regions for quality-neutrals. This figure illustrates the situation in which a larger mass of quality-lovers reside closer to firm B than firm A. Denote the area $C_1 + C_2$ by s_A, which is the proportion of consumers buying product Y_A, i.e., the market share for product Y_A and the area $C_3 + C_4$ by s_B, which is the market share for product Y_B. Algebraically, these market shares are given by:

$$\begin{aligned} s_A &= \text{area}(C_1) + \text{area}(C_2) \\ &= \frac{1}{2}\left(\frac{1-\beta}{2} + \beta\frac{P_B - P_A + \lambda - \theta\tilde{\Delta}}{2\lambda} + \frac{1-\beta}{2}\right)\left(\frac{P_B - P_A + \lambda - \theta\tilde{\Delta}}{2\lambda}\right) \\ &\quad + \frac{1}{2}\left(1 - \frac{1-\beta}{2} + 1 - \beta\frac{P_B - P_A + \lambda}{2\lambda} - \frac{1-\beta}{2}\right)\left(\frac{P_B - P_A + \lambda}{2\lambda}\right) \\ &= \left(1 - \frac{\beta\theta\tilde{\Delta}}{2\lambda}\right)\left(\frac{P_B - P_A + \lambda}{2\lambda}\right) + \frac{\beta\theta^2\tilde{\Delta}^2}{8\lambda^2} - (1-\beta)\frac{\theta\tilde{\Delta}}{4\lambda} \end{aligned} \tag{4}$$

$$\begin{aligned} s_B &= \text{area}(C_3) + \text{area}(C_4) = 1 - s_A \\ &= -\left(1 - \frac{\beta\theta\tilde{\Delta}}{2\lambda}\right)\left(\frac{P_B - P_A + \lambda}{2\lambda}\right) - \frac{\beta\theta^2\tilde{\Delta}^2}{8\lambda^2} + (1-\beta)\frac{\theta\tilde{\Delta}}{4\lambda} + 1, \end{aligned} \tag{5}$$

and the profit functions for each firm, assuming no production costs, are given by:

$$\pi_A = P_A s_A \tag{6}$$

$$\pi_B = P_B s_B \tag{7}$$

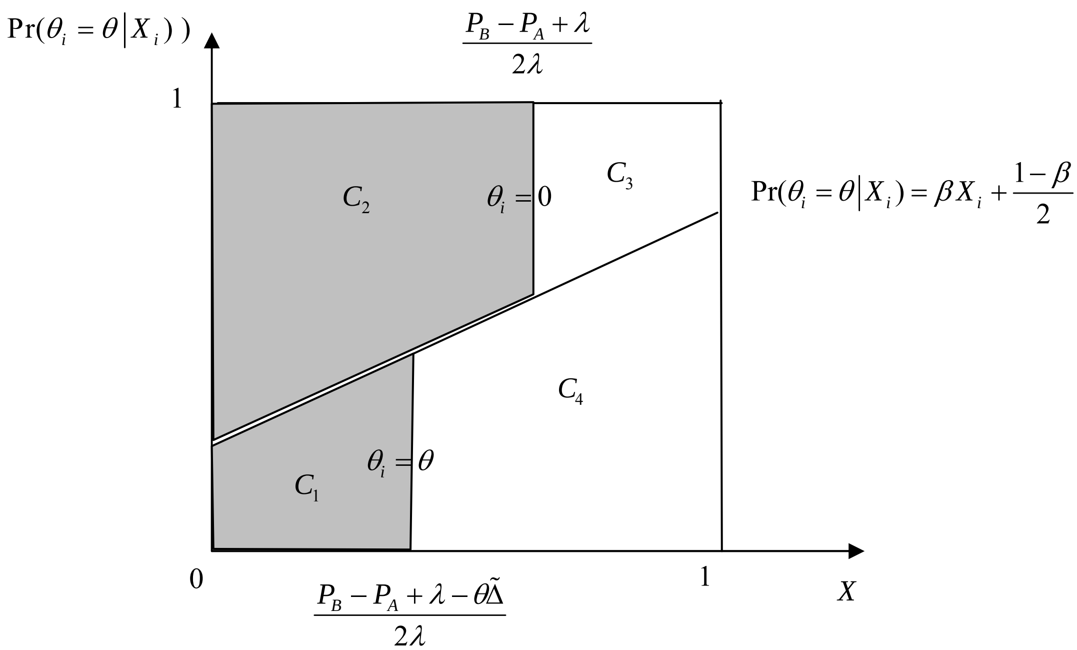

Figure 2: Consumers' Purchase Decisions

To ensure that there are maximal values of profit functions, that prices are strategically complementary, and that there is a unique and stable price equilibrium, we assume that $\theta\Delta < \lambda$. This assumption ensures that the profit functions are well defined. The following first-order conditions characterize the firms' profit maximizing problems:

$$s_A - P_A\left[\frac{1-\dfrac{\beta\theta\tilde{\Delta}}{2\lambda}}{2\lambda}\right]=0, \tag{8}$$

$$s_B - P_B\left[\frac{1-\dfrac{\beta\theta\tilde{\Delta}}{2\lambda}}{2\lambda}\right]=0. \tag{9}$$

The conditions in (8) and (9) imply the following best-response functions:

$$P_A^{BR}=\frac{1}{2}\left(P_B+\lambda+\frac{\beta\theta^2\tilde{\Delta}^2-2\lambda(1-\beta)\theta\tilde{\Delta}}{4\lambda-2\beta\theta\tilde{\Delta}}\right)=\frac{1}{2}\left(P_B+\lambda-\frac{1}{2}\theta\tilde{\Delta}(1-\tilde{\omega})\right) \tag{10}$$

$$P_B^{BR}=\frac{1}{2}\left(P_A+\lambda+\frac{-\beta\theta^2\theta\tilde{\Delta}^2+2\lambda(1+\beta)\theta\tilde{\Delta}}{4\lambda-2\beta\theta\tilde{\Delta}}\right)=\frac{1}{2}\left(P_A+\lambda+\frac{1}{2}\theta\tilde{\Delta}(1+\tilde{\omega})\right), \tag{11}$$

where $\tilde{\omega} = \dfrac{2\lambda\beta}{2\lambda - \beta\theta\tilde{\Delta}}$, and the equilibrium prices for the two products are given by:

$$P_A^* = \lambda + \frac{\theta\tilde{\Delta}}{6}\left(\frac{6\lambda\beta - (2\lambda - \beta\theta\tilde{\Delta})}{2\lambda - \beta\theta\tilde{\Delta}}\right) = \lambda + \frac{\theta\tilde{\Delta}}{2}\left(\tilde{\omega} - \frac{1}{3}\right) \tag{12}$$

$$P_B^* = \lambda + \frac{\theta\tilde{\Delta}}{6}\left(\frac{6\lambda\beta + (2\lambda - \beta\theta\tilde{\Delta})}{2\lambda - \beta\theta\tilde{\Delta}}\right) = \lambda + \frac{\theta\tilde{\Delta}}{2}\left(\tilde{\omega} + \frac{1}{3}\right). \tag{13}$$

It follows from (12) and (13) that:

$$P_A^* + P_B^* = \frac{2\lambda}{\beta}\tilde{\omega}$$
$$P_B^* - P_A^* = \frac{\theta\tilde{\Delta}}{3},$$

and that the equilibrium profits of the two firms are:

$$\Pi_A^* = P_A^* s_A^* = \frac{1}{2}\lambda\left[1 + \frac{\theta\tilde{\Delta}}{2\lambda}\left(\tilde{\omega} - \frac{1}{3}\right)\right]\left[1 + \frac{\theta\tilde{\Delta}}{2\lambda}\left(\frac{\beta\theta\tilde{\Delta}}{6\lambda} - \frac{1}{3}\right)\right] \tag{14}$$

$$\Pi_B^* = P_B^* s_B^* = \frac{1}{2}\lambda\left[1 + \frac{\theta\tilde{\Delta}}{2\lambda}\left(\tilde{\omega} + \frac{1}{3}\right)\right]\left[1 - \frac{\theta\tilde{\Delta}}{2\lambda}\left(\frac{\beta\theta\tilde{\Delta}}{6\lambda} - \frac{1}{3}\right)\right] \tag{15}$$

2.4 Firms' Prices and Profits as Functions of $\tilde{\Delta}$

The following propositions characterize how each firm's prices and profit functions depend on $\tilde{\Delta}$, consumers' perceived difference between the quality of Y_B and Y_A.

<u>Proposition 1</u>: *Firm B's equilibrium price and profit in the second stage are strictly increasing functions of* $\tilde{\Delta}$.

<u>Proof</u>: Consider how firm *B*'s optimal pricing function varies with $\tilde{\Delta}$:

$$\frac{\partial P_B^*}{\partial\tilde{\Delta}} = \frac{\theta}{2}\left(\tilde{\omega}+\frac{1}{3}\right)+\frac{\theta\tilde{\Delta}}{2}\left(\frac{\partial\tilde{\omega}}{\partial\tilde{\Delta}}\right) = \frac{\theta}{2}\left(\tilde{\omega}+\frac{1}{3}\right)+\frac{\theta\tilde{\Delta}}{2}\frac{2\lambda\beta^2\theta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}$$
$$= \frac{\theta}{2}\tilde{\omega}+\frac{\theta}{6}+\frac{\theta^2\tilde{\Delta}}{4\lambda}\frac{4\lambda^2\beta^2}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2} = \frac{\theta}{2}\tilde{\omega}+\frac{\theta}{6}+\frac{\theta^2\tilde{\Delta}}{4\lambda}\tilde{\omega}^2$$
$$= \frac{\theta}{2}\tilde{\omega}\left(1+\frac{\theta\tilde{\Delta}}{2\lambda}\tilde{\omega}\right)+\frac{\theta}{6} = \frac{\theta}{2\beta}\tilde{\omega}^2+\frac{\theta}{6}$$
$$= \frac{\theta}{2}\left[\frac{4\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}+\frac{1}{3}\right]$$

It follows that $\frac{\partial P_B^*}{\partial\tilde{\Delta}} > 0$ because $\theta > 0$, $\beta \geq 0$, and $2\lambda-\beta\theta\tilde{\Delta} > 0$, where the last inequality holds because $0 \leq \beta \leq 1$, $\theta\Delta < \lambda$, and $-\Delta \leq \tilde{\Delta} \leq \Delta$. Firm *B*'s optimal profit function varies with $\tilde{\Delta}$ as follows:

$$\frac{\partial\Pi_B^*}{\partial\tilde{\Delta}} = \frac{\partial P_B^*}{\partial\tilde{\Delta}}s_B^* + P_B^*\frac{\partial s_B^*}{\partial\tilde{\Delta}}$$
$$= \frac{\partial P_B^*}{\partial\tilde{\Delta}}s_B^* + P_B^*\left[\frac{\theta}{4\lambda}(-\frac{\beta\theta\tilde{\Delta}}{6\lambda}+\frac{1}{3})-\frac{\theta\tilde{\Delta}}{4\lambda}\frac{\beta\theta}{6\lambda}\right].$$
$$= \frac{\partial P_B^*}{\partial\tilde{\Delta}}s_B^* + P_B^*\frac{\theta}{12\lambda^2}(\lambda-\beta\theta\tilde{\Delta})$$

It follows that $\frac{\partial\Pi_B^*}{\partial\tilde{\Delta}} > 0$ because $\frac{\partial P_B^*}{\partial\tilde{\Delta}} > 0$, $s_B^* > 0$, $P_B^* > 0$, $\theta > 0$, $\lambda > 0$, and $\lambda-\beta\theta\tilde{\Delta} \geq 0$. ***Q.E.D.***

PROPOSITION 2: *There exists a cutoff value for β, $\beta^* \in (0,1)$, such that when $\beta > \beta^*$, firm A's equilibrium price in the second stage is a strictly increasing function of $\tilde{\Delta}$.*

Proof: Firm *A*'s optimal price varies with $\tilde{\Delta}$ as follows:

$$\frac{\partial P_A^*}{\partial\tilde{\Delta}} = \frac{\theta}{2}(\frac{\tilde{\omega}^2}{\beta}-\frac{1}{3})$$
$$= \frac{\theta}{2}\left[\frac{4\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}-\frac{1}{3}\right]$$

We note that $\frac{\partial P_A^*}{\partial\tilde{\Delta}}$ is not necessarily greater than 0; its value depends on the values of β, λ, θ, and $\tilde{\Delta}$. We can show that $\frac{\partial^2 P_A^*}{\partial\tilde{\Delta}\partial\beta} = \frac{2\theta\lambda^2(2\lambda+\beta\theta\tilde{\Delta})}{(2\lambda-\beta\theta\tilde{\Delta})^3} > 0$, since $\theta > 0$, $\lambda > 0$, $2\lambda+\beta\theta\tilde{\Delta} > 0$, and $2\lambda-\beta\theta\tilde{\Delta} > 0$. The last two inequalities hold because $0 \le \beta \le 1$, $\theta\Delta < \lambda$, and $-\Delta \le \tilde{\Delta} \le \Delta$. Furthermore, when $\beta = 0$, $\frac{\partial P_A^*}{\partial\tilde{\Delta}} = -\frac{\theta}{6} < 0$ and when $\beta = 1$, $\frac{\partial P_A^*}{\partial\tilde{\Delta}} = \frac{\theta}{2}\left[\left(\frac{2\lambda}{2\lambda-\theta\tilde{\Delta}}\right)^2 - \frac{1}{3}\right] \ge \frac{\theta}{2}\left[\left(\frac{2}{3}\right)^2 - \frac{1}{3}\right] > 0$. (Note that $\frac{2\lambda}{2\lambda-\theta\tilde{\Delta}} \ge \frac{2}{3}$ since $2\lambda-\theta\tilde{\Delta} \le 3\lambda$.) Thus, $\frac{\partial P_A^*}{\partial\tilde{\Delta}}$ is strictly increasing in β and goes from negative to positive as β goes from 0 to 1. Thus, there exists a cutoff point $\beta^* \in (0,1)$ such that if $\beta < \beta^*$, $\frac{\partial P_A^*}{\partial\tilde{\Delta}} < 0$, and if $\beta > \beta^*$, $\frac{\partial P_A^*}{\partial\tilde{\Delta}} > 0$. ***Q.E.D.***

<u>PROPOSITION 3</u>: *There exists a cutoff value for β, $\beta^{**} \in (0,1)$, such that when $\beta > \beta^{**}$, firm A's equilibrium level of profits in the second stage is a strictly increasing function of $\tilde{\Delta}$ under one of the two following conditions:*

1) $\tilde{\Delta} \ge 0$, that is, product Y_B is perceived as higher quality than product Y_A,

2) $\tilde{\Delta} < 0$ and $\Delta < K\frac{\lambda}{\theta}$, $K \in (0,1)$, i.e., product Y_B is perceived as lower quality than product Y_A, but the true difference in quality between the two products, $\Delta = q_h - q_l$, is small enough.

<u>Proof</u>: Firm A's optimal profits vary with $\tilde{\Delta}$ in the following way:

$$\begin{aligned}\frac{\partial\Pi_A^*}{\partial\tilde{\Delta}} &= \frac{\partial P_A^*}{\partial\tilde{\Delta}} s_A^* + P_A^* \frac{\partial s_A^*}{\partial\tilde{\Delta}} \\ &= \frac{\theta}{2}\left(\frac{\tilde{\omega}^2}{\beta} - \frac{1}{3}\right) s_A^* - P_A^* \frac{\theta}{12\lambda^2}\left(\lambda - \beta\theta\tilde{\Delta}\right)\end{aligned}$$

$\frac{\partial\Pi_A^*}{\partial\tilde{\Delta}}$ is not greater than 0; its value depends on the values of β, λ, θ, and $\tilde{\Delta}$. However, one can prove that $\frac{\partial^2\Pi_A^*}{\partial\tilde{\Delta}\partial\beta} > 0$. (See Appendix A for a proof.) Furthermore, when $\beta = 0$, $\frac{\partial\Pi_A^*}{\partial\tilde{\Delta}} = -\frac{\theta}{6}s_A^* - P_A^*\frac{\theta}{12\lambda} < 0$.

When $\beta = 1$, $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} > 0$ only if one of the following two conditions holds: (a) $\tilde{\Delta} > 0$ or (b) $\tilde{\Delta} < 0$ and $\Delta < K\frac{\lambda}{\theta}$, $K \in (0,1)$. (See Appendix B for a proof.) That is, under conditions (a) or (b), $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}}$ is strictly increasing in β and goes from negative to positive as β goes from 0 to 1. As such, there exists a cutoff point $\beta^{**} \in (0,1)$ that if $\beta < \beta^*$, $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} < 0$; if $\beta > \beta^*$, $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} > 0$. ***Q.E.D.***

The properties of the second stage prices and profits of firms *A* and *B*, given consumers' beliefs, established in the above propositions play an important role in our analysis of each firm's first stage disclosure decisions. To better understand these properties, we consider how the firms' best-response functions are affected by the correlation between consumer locations and their preferences for quality. When $\beta = 0$, the best-response functions are:

$$P_A^{BR,\beta=0} = \frac{1}{2}\left[P_B + \lambda - \frac{1}{2}\theta\tilde{\Delta}\right] \tag{16}$$

$$P_B^{BR,\beta=0} = \frac{1}{2}\left[P_A + \lambda + \frac{1}{2}\theta\tilde{\Delta}\right], \tag{17}$$

and when $\beta = 1$, the best-response functions are:

$$P_A^{BR,\beta=1} = \frac{1}{2}\left[P_B + \lambda - \frac{1}{2}\theta\tilde{\Delta}\left(1 - \frac{2\lambda}{2\lambda - \theta\tilde{\Delta}}\right)\right] \tag{18}$$

$$P_B^{BR,\beta=1} = \frac{1}{2}\left[P_A + \lambda + \frac{1}{2}\theta\tilde{\Delta}\left(1 + \frac{2\lambda}{2\lambda - \theta\tilde{\Delta}}\right)\right]. \tag{19}$$

In the general case, the best-response functions are a weighted average of those given in (16)-(19):

$$P_A^{BR} = \alpha P_A^{BR,\beta=0} + (1-\alpha)P_A^{BR,\beta=1} \tag{20}$$

$$P_B^{BR} = \alpha P_B^{BR,\beta=0} + (1-\alpha)P_B^{BR,\beta=1}, \tag{21}$$

where $\alpha = 1 - \beta \dfrac{2\lambda - \theta\tilde{\Delta}}{2\lambda - \beta\theta\tilde{\Delta}}$ and $\alpha \in [0,1]$.[21]

Figure 3 illustrates the above decomposition of firms' best-response functions when $\tilde{\Delta} \geq 0$, that is, when consumers perceive that the quality of product Y_B is higher than or equal to that of product Y_A.[22] In Figure 3, the intersection of P_A^{BR} and P_B^{BR} is the price equilibrium. The decomposition shows that any point along the bold line *LH* can be supported as an equilibrium, given that $\alpha \in [0,1]$.[23] Which equilibrium will be realized depends on β. If, for example, $\beta = 1$ (corresponding to $\alpha = 0$), the equilibrium will be at point *H* (the northeast corner of line *LH*). If $\beta = 0$ (corresponding to $\alpha = 1$), the equilibrium will be at point *L* (the southwest corner of line *LH*).

Point *M* is the intersection of best response functions and thus the price equilibrium when $\tilde{\Delta} = 0$, i.e., when consumers perceive the two products to be of the same quality. When consumers perceive that product Y_B is a higher quality product than Y_A, the best response function of firm *B* will always shift upward. The magnitude of this upward shift depends on β, λ, θ, and $\tilde{\Delta}$. The larger β is, the larger is the magnitude of the shift. At the same time, when the value of β is relatively small, firm *A*'s best response function first shifts downward and then shifts upward as β reaches a certain threshold. As shown in Figure 3, the region close to the northeast corner of line *LH* corresponds to the situation where not only firm *B* but also firm *A* is able to charge a higher price when product Y_B is perceived to be of higher quality than Y_A. Suppose consumers already perceive product Y_B as a high-quality product. What would firm *A* do, even if it also has a high-quality product? Figure 3 shows that when β is large enough, both firms would

---

[21] Note that $\dfrac{\partial\alpha}{\partial\beta} = -\dfrac{2\lambda\left(2\lambda - \theta\tilde{\Delta}\right)}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^2} < 0$. When $\beta = 0$, $\alpha = 1$; when $\beta = 1$, $\alpha = 0$. Therefore, $\alpha \in [0,1]$.

[22] The intuition conveyed by Figure 3 is similar to that for the results in Corts (1998), who investigates how price discrimination may lead to an all-worse-off result for firms ranking consumer groups differently.

[23] We have established that $P_B^* - P_A^* = \dfrac{\theta\tilde{\Delta}}{3}$. It follows that the price equilibria is characterized by a straight line that is independent of β and that a point on this line is a weighted average of two end points.

have to lower their prices if firm *A* discloses its high quality. How would this price competition affect firm *A*'s disclosure incentive? Section 2.5 provides an answer.

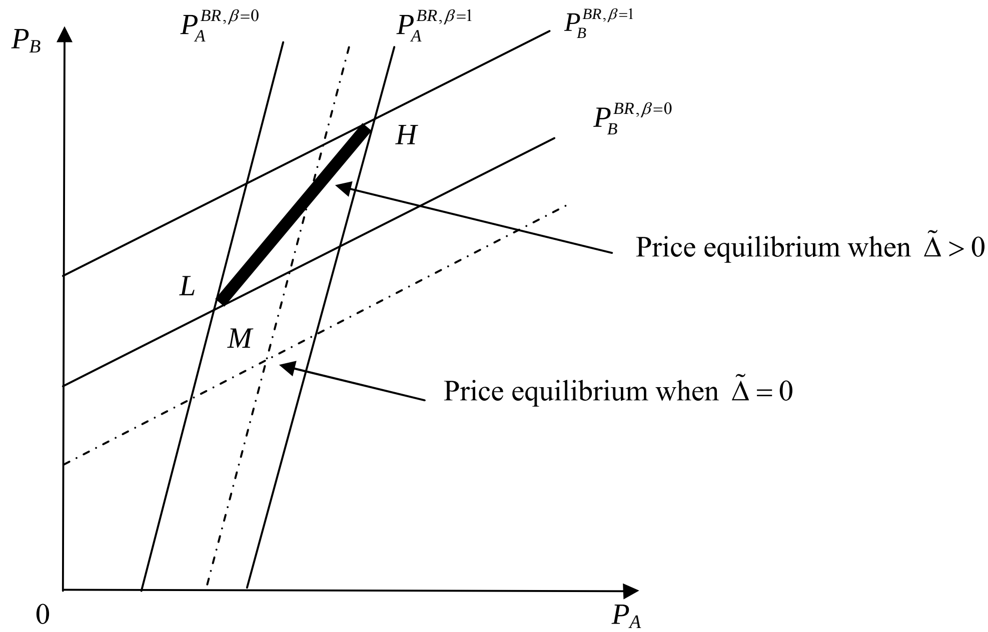

Figure 3: Price Equilibria

2.5 The Unique Perfect Bayesian Equilibrium

PROPOSITION 4: *There exists a cutoff value for* β, $\beta^{**} \in (0,1)$, *such that when* $\beta > \beta^{**}$, *information disclosure is incomplete in the unique Perfect Bayesian Equilibrium of the duopoly game. In this equilibrium:*

1) *Firm B always discloses its quality if it produces a high-quality product.*
2) *If firm B produces (and discloses) a high-quality product, firm A always chooses not to disclose the quality of its product even if it produces a high-quality product. If firm B produces a low-quality product, firm A chooses not to disclose its high quality product if* Δ, *the real quality difference between* Y_A *and* Y_B, *is small enough, i.e.,* $\Delta < K\dfrac{\lambda}{\theta}$, $K \in (0,1)$.
3) *Consumers correctly infer that non-disclosure from firm B means that it has produced a low-quality product. If firm B produces (and discloses) a high-quality product, consumers will perceive the quality difference between the two products to* $\tilde{\Delta}^* = E(Q_B) - E(Q_A) = (1-a_A)\Delta$. *If firm B produces a low-*

quality product and $\Delta < K\frac{\lambda}{\theta}$, $K \in (0,1)$, *consumers will perceive* $\tilde{\Delta}^* = -a_A\Delta$. *Under either circumstance, consumers will maintain their perception of the quality of product* Y_A *as* $E(Q_A) = a_Aq_h + (1-a_A)q_l$.

4) *The equilibrium prices are given by* (12) *and* (13), *and the equilibrium profits are given by* (14) *and* (15) *with* $\tilde{\Delta}^*$ *substituted in for* $\tilde{\Delta}$.

Proof: We consider each of the components of Proposition 4 in turn.

1) Proposition 1 established that firm *B*'s equilibrium profit function in the second stage is strictly increasing in $\tilde{\Delta}$. By disclosing its quality to be high, firm *B* can maximize consumers' perception of $E(Q_B)$ and thus $\tilde{\Delta}$ regardless of whether firm *A* discloses its quality or not. So firm *B*'s dominant strategy, if it produces a high-quality product, is to disclose it.

2) Proposition 3 established that when $\beta > \beta^{**}$, firm *A*'s second-stage equilibrium profit function also is strictly increasing in $\tilde{\Delta}$ if either condition 1) ($\tilde{\Delta} \geq 0$) or 2) ($\tilde{\Delta} < 0$ and $\Delta < K\frac{\lambda}{\theta}$, $K \in (0,1)$) holds.

   There are two cases to consider with respect to firm *A*'s disclosure decision:

   a) If firm *B* produces (and discloses) a high-quality product, we will have $\tilde{\Delta} \geq 0$ since $E(Q_A) \leq E(Q_B) = q_h$. If firm *A* also produces and discloses a high-quality product, consumers will perceive $\tilde{\Delta} = 0$. As a result, firm *A* would like to maximize consumers' perception of $\tilde{\Delta}$ by not disclosing the fact that it has a high-quality product. (Recall that given the assumption made about the disclosure technology, neither firm will be able to disclose its quality if it produces a low-quality product.)

   b) If firm *B* produces a low-quality product and the true difference between the high- and low-quality products, Δ, is small enough—where "small enough" is that $\Delta < K\frac{\lambda}{\theta}$, $K \in (0,1)$—consumers will perceive $\tilde{\Delta} < 0$ since $E(Q_A) \geq E(Q_B) = q_l$. If firm *A* produces and discloses a high-quality product, consumers will perceive $\tilde{\Delta} = -\Delta$, where $-\Delta$ is the lower bound for $\tilde{\Delta}$. In this situation, firm A will again increase $\tilde{\Delta}$ by not disclosing it has a high-quality product.

3) Consumers update their beliefs using Bayes' rule based on what either firm does (or does not) disclose about the quality of its product. It is straightforward to establish that consumers associate low quality with undisclosed product Y_B because firm B's dominant strategy is to disclose it is producing a high-quality product. When the quality of Y_A is undisclosed, consumers gain no new information about this product if firm B produces a high-quality product or if firm B produces a low-quality product but $\Delta < K\frac{\lambda}{\theta}$, $K \in (0,1)$. Under either circumstance, as consumers know that firm A will not choose disclosure no matter what firm B does, we have

$$\begin{aligned}&\Pr\left(Q_A = q_h \,|\, A:\text{no disclosure}\right)\\ &\quad= \frac{\Pr\left(A:\text{no disclosure}\,|\,Q_A = q_h\right)\Pr\left(Q_A = q_h\right)}{\Pr\left(A:\text{no disclosure}\,|\,Q_A = q_h\right)\Pr\left(Q_A = q_h\right) + \Pr\left(A:\text{no disclosure}\,|\,Q_A = q_l\right)\Pr\left(Q_A = q_l\right)}\\ &\quad= \frac{a_A}{a_A + \left(1 - a_A\right)} = a_A\end{aligned}$$

This means, consumers will maintain their priors about the quality of product Y_A when there is no disclosure from firm A.

4) With consumers' beliefs as specified in 3), neither firm will want to deviate from their strategies specified in 1) and 2). It is then straightforward to calculate the equilibrium prices and profits.

The above logic establishes that the Perfect Bayesian Equilibrium is unique as either firm has its dominant disclosure and pricing strategy for a given set of parameter values and for consumers' Bayesian-updated beliefs. Under conditions specified in this proposition, information disclosure is *incomplete*, in that firm A will always choose to not disclose the quality of its product, even if it produces a high-quality one. ***Q.E.D.***

It is worth emphasizing that there are no other consumer beliefs in response to non-disclosure that would induce firm A to disclosure the fact that it has a high-quality product. For suppose that consumers were to believe that Y_A is definitely low-quality when firm A chooses not to disclose the quality of its product. Under the conditions specified in Proposition 4, firm A still benefits from not disclosing that it has a high-quality product because, in doing so, it induces a higher value of $\tilde{\Delta}$—and its profit-reducing consequences—than by disclosing.

To summarize, we have shown, via a simple model, that firms' disclosure incentives, as well as market outcomes under different information scenarios, critically hinges on the nature of consumer heterogeneity across the two attributes of products—consumer location and the quality of firms' products—where this heterogeneity depends on the values of β, λ, θ, and Δ. When $\beta < \beta^{**}$ the "unraveling result" of Grossman and Milgrom holds as either firm with a high-quality product will find it in their interest to disclose this fact. However, when $\beta > \beta^{**}$, "unraveling" with respect to disclosure of the quality of firms' products breaks down.

3. Intuition and Implications

As is well known, the amount of information consumers have about the attributes of differentiated products affects their consumption decisions. Full information allows consumers to assess the entire bundle of attributes contained in each product and, as a result, consumers' choices can exactly reflect their willingness to pay for each product. However, allowing consumers to make such assessment, under certain circumstances, may not always be in the interests of firms. More information may change the substitutability of products and make consumers more sensitive to the prices charged by firms, which firms want to avoid. In this section, we provide some further intuition about the possibility of incomplete disclosure equilibrium established in Proposition 4 by relating firms' strategic disclosure decisions to how disclosure can change the elasticities of demand for multi-attribute products.

3.1 Product Substitution Patterns and Demand Elasticities

Our results indicate that whether complete disclosure increases the price responsiveness of consumers for a product depends relies on how consumers' tastes for quality and their locations vis-à-vis firms are jointly distributed. To develop this relationship, we need to characterize the elasticities of demand for products Y_A and Y_B. Consider first the demand elasticities for the two products under consumer beliefs $\tilde{\Delta}$. The demand elasticities for products Y_B and Y_A are given by:

$$
\begin{aligned}
e_B &\equiv -\frac{\partial s_B}{\partial P_B}\frac{P_B}{s_B} \\
&= \frac{\frac{P_A}{2\lambda}\left(1-\frac{\beta\theta\tilde{\Delta}}{2\lambda}\right)}{-\left(1-\frac{\beta\theta\tilde{\Delta}}{2\lambda}\right)\left(\frac{P_B-P_A+\lambda}{2\lambda}\right)-\frac{\beta\theta^2\tilde{\Delta}^2}{8\lambda^2}+(1-\beta)\frac{\theta\tilde{\Delta}}{4\lambda}}, \qquad (22) \\
&= \frac{P_A}{-(P_B-P_A+\lambda)-\frac{\theta\tilde{\Delta}}{2}(\tilde{\omega}-1)+\frac{2\lambda\tilde{\omega}}{\beta}}
\end{aligned}
$$

and

$$
\begin{aligned}
e_A &\equiv -\frac{\partial s_A}{\partial P_A}\frac{P_A}{s_A} \\
&= \frac{\frac{P_A}{2\lambda}\left(1-\frac{\beta\theta\tilde{\Delta}}{2\lambda}\right)}{\left(1-\frac{\beta\theta\tilde{\Delta}}{2\lambda}\right)\left(\frac{P_B-P_A+\lambda}{2\lambda}\right)+\frac{\beta\theta^2\tilde{\Delta}^2}{8\lambda^2}-(1-\beta)\frac{\theta\tilde{\Delta}}{4\lambda}}. \qquad (23) \\
&= \frac{P_A}{(P_B-P_A+\lambda)+\frac{\theta\tilde{\Delta}}{2}(\tilde{\omega}-1)},
\end{aligned}
$$

respectively. One can prove that e_B is strictly decreasing in $\tilde{\Delta}$, i.e., $\frac{\partial e_B}{\partial \tilde{\Delta}} < 0$. (See appendix C.) One also can prove that e_A is strictly decreasing in $\tilde{\Delta}$, i.e., $\frac{\partial e_A}{\partial \tilde{\Delta}} < 0$, when β is above a threshold in the range of (0,1). (See Appendix D.) It follows that when β is large enough, price elasticities will increase for both firms as $\tilde{\Delta}$ decreases and will decrease as $\tilde{\Delta}$ increases. Higher demand elasticities imply a greater degree of substitutability among products, which results in more intensive price competition among firms. As we have noted above, revelation of an attribute of a product to consumers to decrease $\tilde{\Delta}$ will result in a decrease in prices and profits for both firms. As Proposition 4 establishes, a firm (in this case, firm *A*) will tend to find it in its interest to not disclosure the unobserved attribute of its product in order to avoid inducing a reduction in $\tilde{\Delta}$, the perceived difference in the quality of the products available to consumers. To do otherwise would induce greater price competition with its rival and, in the end, reduce its profits.

There is another way to view the interaction between the heterogeneity of consumers across the attributes of multi-attribute products and its consequences for firms' strategic behavior. Notice that as β gets larger, consumers systematically become more "mismatched" with respect to their location and tastes for quality when firm *A* produces a high-quality product and firm *B* produces a low-quality product. As β approaches 1, quality-lovers, on average, are located closer to the low-quality product (sold by firm *B*) and quality-neutrals are located closer to the high-quality product (sold by firm *A*). That is, the "home" market for the low-quality product is disproportionately made up of quality-lovers and visa-versa. If firm *A* discloses that it has a high quality, consumers will be able to fully recognize this mismatch and, as a result, will be less willing to pay as much for products close to them than they were prior to firm *A*'s disclosure. They will become more price-sensitive and want to be compensated for the mismatch in location and quality, thereby forcing firms to lower their prices. A similar intuition can be developed about the case when both firms produce high-quality products. In this case, consumers and products also are mismatched, although to a lesser degree. If the quality of products is revealed to consumers, firm *B* will lose its dominance with quality-lovers who are located close to it when firm *A* also has a high-quality product.

As is well known from the literature on product differentiation, firms have strategic incentives to differentiate their products to reduce demand elasticities of their products and alleviate price competition with rivals selling otherwise similar products. Disclosing information on product quality can precisely achieve the effect of differentiating a firm's own product away from that of its competitor when the unobserved quality is the single dimension of product differentiation. However, this is not true when there preexists another product attribute. In our model, from the perspective of consumers, both products are fully differentiated by their locations, as the two firms are located at opposite ends of the linear city. Even with no information about product quality, each firm already faces a downward sloping demand curve for its product. Consumers do not view the two products as perfect substitutes and firms selling such differentiated products earn positive profits. With the existing horizontal differentiation, a firm's incentive to differentiate its product along another dimension via disclosure will be complicated and strategically driven. Nothing concerning firms' proprietary information can be so readily "unraveling": a firm producing a product with a high-quality attribute may or may not reveal this attribute, depending on whether the revelation gives it the necessary "niche."

3.2 An Illustration

In this section, we illustrate how our results might play out with an example.[24] Suppose there are two restaurants in a market, one is a fast food outlet specializing in hamburgers and the other is a French restaurant featuring fancy dishes like escargot. Two types of consumers—students and professors—populate this market. At the same price for a meal, we suppose that, on average, students strongly prefer hamburgers to French cuisine as eating fast food saves time for studying, while professors, who have more sophisticated palates, have just the opposite tastes. Meanwhile, professors also have higher hygiene standards than do students.[25]

Suppose the two restaurants differ in their hygienic practices. Suppose, in fact, that the French restaurant maintains a very sanitary kitchen to cater to professors' standards, while the fast food restaurant's hygienic practices are more lax. Neither type of consumers, on their own, can readily determine the hygienic quality of the restaurants, although they have formed priors based on past experience. To reduce health risks, the local public health department inspects restaurants and rates their hygiene quality. Suppose that the department, based on its inspections, awards "clean-kitchen" certificates to restaurants above a certain hygiene quality but that it is up to the restaurants as to whether they reveal to the general public whether they have met this threshold by posting the certificates at the entrance of their restaurants.[26] The

[24] This example is inspired by Jin and Leslie (2003), who study the hygiene practices and sales volume of restaurants in Los Angeles County under alternative disclosure requirements concerning the findings of public health inspections.

[25] Professors, say, cannot afford to miss the classes they teach due to food poisoning while students do not mind missing a few of those classes, even if they have to spend the time in the student infirmary!

[26] For example, the Los Angeles County Department of Public Health issued hygiene "grade cards" to restaurants and, in some areas of the county, allowed restaurants the choice of whether or not to display them.

inspections are unannounced and the inspectors rate the hygiene conditions on the day of inspection. Since hygiene conditions can vary over time, a more sanitary kitchen can sometimes fail to obtain certification and a usually dirty kitchen can sometimes pass an inspection.

Our model indicates that firms' disclosure incentives depend on the distribution of consumer tastes for the two types of cuisine and for hygiene quality. Recall that professors, on average, strongly prefer French cuisine to fast food while students' preferences over types of cuisine are just the opposite. Also recall that consumers' priors are that the French restaurant maintains a more sanitary kitchen. Let's first consider the case that the French restaurant has a bad day when inspected while the fast food outlet has a good one and receives a certificate. While professors prefer French cuisine to fast food, they also prefer better hygiene—something that the French restaurant currently lacks. However, burger-loving students do not place value on better hygienic practices, at which the fast food outlet currently seems to excel. Will the fast food outlet want to post its "clean kitchen" certificate?

Suppose the fast food restaurant does post its certificate. Naturally, some professors, who value French food less strongly than other professors (and/or who value hygiene quality more strongly than others), want to switch to the fast food outlet upon learning that it maintains more sanitary conditions. In response, the French restaurant may want to mark down the price it charges for a meal to lure back its consumer base. This move, however, will affect the fast food outlet's market share, as some students, who value hamburgers less strongly than other students, will be willing to switch to the French restaurant for cheaper meals. In turn, the fast food outlet may also want to reduce the price of its hamburgers, which may induce further price-cutting by the French restaurant. As a result of this price competition, both restaurants would end up with lower prices and profits if the fast food outlet were to post its certificate. Thus disclosing its hygiene rating would not be in the fast food outlet's self-interest. As a result, voluntary disclosure will not occur in this market, even though both professors and students may be better off by having information on hygiene quality—for different reasons—prior to deciding where to dine.

Consider another scenario. Suppose both restaurants have a good day when inspected and are both are awarded certificates for acceptable hygiene quality. If these two restaurants were mandated to display their certificates, the food each serves would become more substitutable for professors and, as a result, professors would have an incentive to switch restaurants. The French restaurant would then have an incentive to lower its prices to retain its professor clientele, and the price wars described above would again follow. Both firms would potentially lose from this escalation of competition and, as a result, one or both restaurants have an incentive to refrain from posting its certificate.

What is central to this example is that consumers value both the horizontal attribute (taste of food, or time saving) and a vertical attribute (hygiene quality) of the product (a meal outside the home) in a market in which buyer-seller sorting has resulted in a particular form of segregation of consumers. If consumers know both attributes of the meals each restaurant is serving, they will choose to trade-off their preferred meals for more sanitary food and need to be compensated for poorer hygiene or meals which they enjoy less. Under this configuration of consumer preferences, firms may not have an incentive to break the existing taste-segregation of consumers by disclosing information about one of the attributes of their meals. Disclosure may increase the substitutability between the two types of meals, intensify price competition, and lower profits for both restaurants. This makes disclosure undesirable for a restaurant serving sanitary food.

4. Conclusion

Effective provision of information ensures the efficiency of market operations and benefits social welfare in a variety of ways. Laws and regulations that make disclosure of information about products is one way to insure that consumers have such information. If, however, the “unraveling result” of Grossman (1981) and Milgrom (1981) holds, mandatory disclosure is unnecessary since firms have sufficient incentives to voluntarily disclose the quality of their products, so long as disclosure is verifiable and has trivial costs. As a result, we should not expect to see any systematic change in disclosure behaviors, prices, and profits of firms when disclosure mechanisms—either from voluntary to mandatory or vise-versa—change.

The findings in this paper cast doubt on the above characterization of the potential need for non-market forces to ensure disclosure about product quality. In particular, we have shown that the unraveling result of Grossman and Milgrom can break down when products have more than one attribute and there is heterogeneity in consumer tastes over these attributes. We are able to show that firms do not always have full incentives to voluntarily disclose information on product quality as providing more information to consumers can cause more elastic demands and, thus, can intensify price competition among firms. As a result, government intervention in the form of mandatory disclosure laws may change firm behavior and benefit consumers in markets with incomplete information. In fact, several recent empirical investigations of particular markets (Mathios, 2000; Jin and Leslie, 2003) has shown that mandatory disclosure laws do make a difference, indicating that there may be financial gain to firms selling differentiated products from maintaining "a culture of silence" with respect to the true quality of their products.[27] More accurately, we show that mandatory disclosure laws may reduce prices in certain product markets as the resulting disclosure of information intensifies price competition. In principle, this latter implication is testable by examining how prices for certain products change before and after a change in a disclosure regime, such as the imposition of a law mandating the disclosure of attributes of a product.

More importantly, we find what happens to prices, profits, and consumer welfare under different information scenarios depends on the distribution of consumer preferences over all of the attributes of products. In effect, disclosure of product quality by some firms can exert an "externality" effect on their rivals which can benefit consumers but make firms worse off. Without non-market forces, such as governments mandating disclosure, firms may find that their self-interest does not lie with disclosing information about product quality that the "the unraveling result" implies.

[27] Jin and Leslie (2003) note: "One may wonder why restaurants did not disclose the results of their hygiene inspections prior to the grade cards. Why would a restaurant manager not create their own poster clearly showing their latest hygiene score, say, and display it in the window? Perhaps this indicates it is unprofitable for restaurants to increase the provision of hygiene quality information to consumers."

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Appendix

A) Proof of $\frac{\partial^2 \Pi_A^*}{\partial \tilde{\Delta} \partial \beta} > 0$ in Proposition 3:

<u>Proof</u>:

$$\frac{\partial^2 \Pi_A^*}{\partial \tilde{\Delta} \partial \beta} = \frac{\partial^2 P_A^*}{\partial \tilde{\Delta} \partial \beta} s_A^* + \frac{\partial P_A^*}{\partial \tilde{\Delta}} \frac{\partial s_A^*}{\partial \beta} + \frac{\partial P_A^*}{\partial \beta} \frac{\partial s_A^*}{\partial \tilde{\Delta}} + P_A^* \frac{\partial^2 s_A^*}{\partial \tilde{\Delta} \partial \beta}$$

$$= \frac{2\theta\lambda^2 \left(2\lambda + \beta\theta\tilde{\Delta}\right)}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^3} \left[\frac{1}{2} - \frac{\theta\tilde{\Delta}\left(2\lambda - \beta\theta\tilde{\Delta}\right)}{24\lambda^2} \right] + \frac{\theta}{2}\left(\frac{\omega^2}{\beta} - \frac{1}{3} \right) \frac{\theta^2 \tilde{\Delta}^2}{24\lambda^2}$$

$$+ \frac{\theta\tilde{\Delta}}{2} \frac{4\lambda^2}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^2} \frac{\theta}{4\lambda} \left(\frac{\beta\theta\tilde{\Delta}}{3\lambda} - \frac{1}{3} \right) + \left[\lambda + \frac{\theta\tilde{\Delta}}{2}\left(\omega - \frac{1}{3} \right) \right] \frac{\theta^2 \tilde{\Delta}}{12\lambda^2}$$

$$= \frac{\theta\lambda^2 \left(2\lambda + \beta\theta\tilde{\Delta}\right)}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^3} - \frac{\theta^2 \tilde{\Delta}\left(2\lambda + \beta\theta\tilde{\Delta}\right)}{12\left(2\lambda - \beta\theta\tilde{\Delta}\right)^2} + \frac{\theta^3 \tilde{\Delta}^2}{12} \left[\frac{\beta}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^2} - \frac{1}{12\lambda^2} \right]$$

$$- \frac{\theta^2 \tilde{\Delta}\left(\lambda - \beta\theta\tilde{\Delta}\right)}{6\left(2\lambda - \beta\theta\tilde{\Delta}\right)^2} + \frac{\theta^2 \tilde{\Delta}}{6} \frac{12\lambda^2 - \theta\tilde{\Delta}\left(2\lambda - \beta\theta\tilde{\Delta}\right)}{12\lambda^2 \left(2\lambda - \beta\theta\tilde{\Delta}\right)}$$

$$= \frac{\theta\lambda^2 \left(2\lambda + \beta\theta\tilde{\Delta}\right)}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^3} - \frac{\lambda\theta^2 \tilde{\Delta}}{6\left(2\lambda - \beta\theta\tilde{\Delta}\right)^2} - \frac{\theta^3 \tilde{\Delta}^2}{12(12\lambda^2)} + \frac{\theta^2 \tilde{\Delta}}{6} \left[\frac{\lambda}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^2} - \frac{\theta\tilde{\Delta}}{12\lambda^2} \right]$$

$$= \frac{\theta\lambda^2 \left(2\lambda + \beta\theta\tilde{\Delta}\right)}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^3} - \frac{\theta^3 \tilde{\Delta}^2}{48\lambda^2}$$

$$= \theta \left[\frac{\lambda^2 \left(2\lambda + \beta\theta\tilde{\Delta}\right)}{\left(2\lambda - \beta\theta\tilde{\Delta}\right)^3} - \frac{\theta^2 \tilde{\Delta}^2}{48\lambda^2} \right]$$

$$> \theta\left(\frac{1}{27} - \frac{1}{48}\right) > 0$$

The inequalities hold because $\theta > 0$, $\lambda > 0$, $0 \le \beta \le 1$, $\theta\Delta < \lambda$, and $-\Delta \le \tilde{\Delta} \le \Delta$ so that $\frac{\lambda}{2\lambda - \beta\theta\tilde{\Delta}} > \frac{1}{3}$ and $\frac{\theta^2 \tilde{\Delta}^2}{\lambda^2} < 1$. ***Q.E.D.***

B) Proof of $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} > 0$ if $\beta = 1$ and one of the two following conditions hold: 1) $\tilde{\Delta} \ge 0$ or 2) $\tilde{\Delta} < 0$ and $\Delta < K\frac{\lambda}{\theta}$, $K \in (0,1)$ in Proposition 3:

Proof:

$$\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} = \frac{\theta}{2}\left[\frac{4\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}-\frac{1}{3}\right]\left[\frac{1}{2}+\frac{\theta\tilde{\Delta}}{4\lambda}\left(\frac{\beta\theta\tilde{\Delta}}{6\lambda}-\frac{1}{3}\right)\right]-\left[\lambda+\frac{\theta\tilde{\Delta}}{2}\left(\frac{2\lambda\beta}{2\lambda-\beta\theta\tilde{\Delta}}-\frac{1}{3}\right)\right]\frac{\theta}{12\lambda^2}\left(\lambda-\beta\theta\tilde{\Delta}\right)$$

$$=\frac{\theta}{6}\left[\frac{6\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}-\frac{\beta\theta\tilde{\Delta}}{2\left(2\lambda-\beta\theta\tilde{\Delta}\right)}-\frac{1}{2}+\frac{\theta\tilde{\Delta}}{24\lambda^2}\left(2\lambda-\beta\theta\tilde{\Delta}\right)\right]-\frac{\theta}{6}\left[\lambda+\frac{\theta\tilde{\Delta}}{6}\left(\frac{6\lambda\beta-2\lambda+\beta\theta\tilde{\Delta}}{2\lambda-\beta\theta\tilde{\Delta}}\right)\right]\frac{\lambda-\beta\theta\tilde{\Delta}}{2\lambda^2}$$

$$=\frac{\theta}{6}\left[\frac{6\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}-\frac{\lambda}{2\lambda-\beta\theta\tilde{\Delta}}+\frac{\theta\tilde{\Delta}}{24\lambda^2}\left(2\lambda-\beta\theta\tilde{\Delta}\right)-\frac{\lambda-\beta\theta\tilde{\Delta}}{2\lambda}-\frac{\theta\tilde{\Delta}\left(\lambda-\beta\theta\tilde{\Delta}\right)}{12\lambda^2}\left(\frac{6\lambda\beta-2\lambda+\beta\theta\tilde{\Delta}}{2\lambda-\beta\theta\tilde{\Delta}}\right)\right]$$

$$=\frac{\theta}{6}\left[\frac{\lambda\left(6\lambda\beta-2\lambda+\beta\theta\tilde{\Delta}\right)}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}-\frac{\theta\tilde{\Delta}\left(\lambda-\beta\theta\tilde{\Delta}\right)}{12\lambda^2}\left(\frac{6\lambda\beta-2\lambda+\beta\theta\tilde{\Delta}}{2\lambda-\beta\theta\tilde{\Delta}}\right)+\frac{\theta\tilde{\Delta}}{24\lambda^2}\left(2\lambda-\beta\theta\tilde{\Delta}\right)-\frac{\lambda-\beta\theta\tilde{\Delta}}{2\lambda}\right]$$

$$=\frac{\theta}{6}\left[\left(\frac{6\lambda\beta-2\lambda+\beta\theta\tilde{\Delta}}{12\lambda^2\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}\right)\left[12\lambda^3-\theta\tilde{\Delta}\left(\lambda-\beta\theta\tilde{\Delta}\right)\left(2\lambda-\beta\theta\tilde{\Delta}\right)\right]+\frac{\theta\tilde{\Delta}\left(2\lambda-\beta\theta\tilde{\Delta}\right)-12\lambda\left(\lambda-\beta\theta\tilde{\Delta}\right)}{24\lambda^2}\right]$$

$$=\frac{\theta}{6}\left[\frac{144\lambda^4\beta-96\lambda^4+96\lambda^3\beta\theta\tilde{\Delta}+16\lambda^3\theta\tilde{\Delta}-28\lambda^2\beta\theta^2\tilde{\Delta}^2-24\lambda^2\beta^2\theta^2\tilde{\Delta}^2+16\lambda\beta^2\theta^3\tilde{\Delta}^3-3\beta^3\theta^4\tilde{\Delta}^4}{24\lambda^2\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}\right]$$

$$=\frac{\theta}{6}\left[\frac{144\lambda^4\beta-3\beta\theta^2\tilde{\Delta}^2\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2-4\lambda\left(6\lambda-\theta\tilde{\Delta}\right)\left(2\lambda-\beta\theta\tilde{\Delta}\tilde{\Delta}\right)^2}{24\lambda^2\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}\right]$$

$$=\theta\left[\frac{\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}-\frac{\beta\theta^2\tilde{\Delta}^2}{48\lambda^2}-\frac{\left(6\lambda-\theta\tilde{\Delta}\right)}{36\lambda}\right]$$

When $\beta = 1$, $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} = \theta\left[\frac{\lambda^2}{\left(2\lambda-\theta\tilde{\Delta}\right)^2}-\frac{\theta^2\tilde{\Delta}^2}{48\lambda^2}-\frac{6\lambda-\theta\tilde{\Delta}}{36\lambda}\right]$. If $\tilde{\Delta} \geq 0$, $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} > \theta(\frac{1}{4}-\frac{1}{48}-\frac{1}{6}) > 0$ because $\theta >$ 0, $\lambda > 0$, $\theta\Delta < \lambda$, and $0 \leq \tilde{\Delta} \leq \Delta$ so that $\frac{\lambda}{2\lambda-\theta\tilde{\Delta}} > \frac{1}{2}$, $\frac{\theta^2\tilde{\Delta}^2}{\lambda^2} < 1$, and $\frac{6\lambda-\theta\tilde{\Delta}}{\lambda} < 6$. If $\tilde{\Delta} < 0$, we can show that $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}}$ is strictly increasing in $\frac{\theta\tilde{\Delta}}{\lambda}$. When $\frac{\theta\tilde{\Delta}}{\lambda} = -1$, $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} < -1$. When $\frac{\theta\tilde{\Delta}}{\lambda} = 0$, $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} > 0$. That is, when $\frac{\theta\tilde{\Delta}}{\lambda}$ is above a constant $-K \in [-1,0]$ we will have $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} > 0$. This condition implies $\tilde{\Delta} > -K\frac{\lambda}{\theta}$. Because $-\Delta < \tilde{\Delta} < 0$—so long as $\Delta < K\frac{\lambda}{\theta}$, $K \in (0,1)$—we will have $\tilde{\Delta} > K\frac{\lambda}{\theta}$. Thus $\frac{\partial \Pi_A^*}{\partial \tilde{\Delta}} > 0$. ***Q.E.D.***

C) Proof of $\frac{\partial e_B}{\partial \tilde{\Delta}} < 0$ in Section 3.1:

Proof:

$$\frac{\partial e_B}{\partial \tilde{\Delta}} = -\frac{P_B\left[-\frac{\theta}{2}\left(\frac{2\lambda\beta}{2\lambda-\beta\theta\tilde{\Delta}}-1\right)-\frac{\theta\tilde{\Delta}}{2}\frac{2\lambda\beta^2\theta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}+\frac{4\lambda^2\beta\theta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}\right]}{\left[-\left(P_B-P_A+\lambda\right)-\frac{\theta\tilde{\Delta}}{2}(\tilde{\omega}-1)+\frac{2\lambda\tilde{\omega}}{\beta}\right]^2}$$

$$= \frac{-P_B\frac{\theta}{2}\left[\frac{4\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}+1\right]}{\left[-\left(P_B-P_A+\lambda\right)-\frac{\theta\tilde{\Delta}}{2}(\tilde{\omega}-1)+\frac{2\lambda\tilde{\omega}}{\beta}\right]^2} < 0$$

The inequality holds because $\theta > 0$, $0 \le \beta \le 1$, and $2\lambda - \beta\theta\tilde{\Delta} > 0$. ***Q.E.D.***

D) Proof of $\frac{\partial e_A}{\partial \tilde{\Delta}} < 0$ when β is large enough in Section 3.1:

Proof:

$$\frac{\partial e_A}{\partial \tilde{\Delta}} = -\frac{P_A\left[\frac{\theta}{2}\left(\frac{2\lambda\beta}{2\lambda-\beta\theta\tilde{\Delta}}-1\right)+\frac{\theta\tilde{\Delta}}{2}\frac{2\lambda\beta^2\theta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}\right]}{\left[\left(P_B-P_A+\lambda\right)+\frac{\theta\tilde{\Delta}}{2}(\tilde{\omega}-1)\right]^2}$$

$$= -\frac{P_A\frac{\theta}{2}\left[\left(\frac{4\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}-1\right)\right]}{\left[\left(P_B-P_A+\lambda\right)+\frac{\theta\tilde{\Delta}}{2}(\tilde{\omega}-1)\right]^2}$$

We can easily show that when β is above a threshold in the range of (0,1), $\frac{4\lambda^2\beta}{\left(2\lambda-\beta\theta\tilde{\Delta}\right)^2}-1 > 0$ and thus

$\frac{\partial e_A}{\partial \tilde{\Delta}} < 0$. ***Q.E.D.***