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**Preferences and Pathways to
Segregation: Reply to Van de Rijt,
Siegel, and Macy**

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**PREFERENCES AND PATHWAYS TO SEGREGATION:
REPLY TO VAN DE RIJT, SIEGEL, AND MACY**

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INTRODUCTION

We are pleased that Van de Rijt, Siegel, and Macy (VSM) have taken an interest in our work. Since the publication of our article (Bruch and Mare 2006) we too have examined the role of random error in segregation dynamics and formally examined the relationship between residential preferences and segregation (Mare 2007; Tuljapurkar, Bruch, and Mare 2008). We welcome the opportunity to discuss these issues, compare our conclusions to those of VSM, and extend our 2006 argument regarding the preferences of individuals and the dynamics of residential segregation. We also acknowledge and present corrections of errors in our 2006 article. In preparing our software for public release, we found an error in our computer code. Our corrected results show that, as VSM point out, some of continuous functions for individuals' decisions about whether and where to move that we originally claimed to generate integration in fact lead to segregation. Our original findings regarding continuous functions with varying β parameters (Bruch and Mare 2006:692) were wrong. This reply includes corrected versions of our simulations. However, the error in our code notwithstanding, our original conclusions regarding the effects of the form of individual preferences on segregation dynamics are correct.

Our corrected software---including an executable file, the open-source Java code, and a suite of testing software for verifying key features of agent-based models---is publically available.¹ Our software can be extended to look at various dynamic processes (for example, marriage markets, peer effects, and the spread of innovation) and we encourage interested researchers to build upon our source code.

Our 2006 article reported an investigation of the links between how people evaluate neighborhoods and aggregate segregation dynamics. We emphasized that the form of residential choice functions (that is, how individuals evaluate and choose neighborhoods) has important implications for segregation dynamics. Simulations based on our corrected code show that, as VSM report, hypothetical monotonic, continuous functions with a sufficiently strong response to the racial makeup of a neighborhood (the coefficient β in the functions that describe residential choice) can generate segregation, and that empirical preference functions based on Detroit Area Study (DAS) data are consistent with high segregation. However, our argument that “regions of indifference” across neighborhoods with varying ethnic composition (a key feature of threshold functions) play an important role in segregation dynamics still holds. Moreover, as we show below, the shape of residential preference functions affects segregation dynamics through other pathways as well.

Our reply first summarizes our ideas about preference functions and the effect of random variation on segregation dynamics, and then responds more directly to VSM’s comment. We review the ways that randomness enters into choice processes and argue that the shape of residential choice function -- for example, whether preferences follow a continuous or a threshold function -- affects segregation dynamics through three

¹ All software and associated files are GPL licensed and open-source. Software can be downloaded at <http://www-personal.umich.edu/~ebruch/software/software.html>.

pathways: the baseline level of randomness in the choice process, how random error fades out or cumulates over time, and the speed with which integrated communities converge to an equilibrium level of segregation. Our corrected agent-based models produce the same patterns as the ones shown by VSM and we agree with them that continuous functions with a sufficiently high β can produce segregation. However, we believe that VSM's claim that "sensitivity to chance" matters more than "sensitivity to change" is misleading because these are not separable dimensions of the choice function. Rather, these factors interact in a complex yet interpretable way. We also show that, contrary to VSM's claims, continuous functions with a sufficiently low randomness do *not* result in higher levels of segregation than threshold functions, although continuous functions do reach equilibrium more rapidly. We also explain an important feature of VSM's Figures C1 and C2, namely that, above a certain β value, the threshold functions appear less responsive than continuous functions to increases in β .

We then address the issue, raised by VSM, of how preferences for racial *integration* affect segregation. We find VSM's argument regarding "the paradox of strong versus weak preferences" unpersuasive. VSM argue that stronger preferences for integration result in higher segregation when residential choice follows a continuous rather than threshold function, but we show that this conclusion is an artifact of VSM's highly stylized specification of these functions. VSM's "paradox" occurs only under a narrow set of assumptions. We show that preferences for integration expressed by black respondents in the DAS are consistent with very low segregation and offer a more plausible statement about the link between preferences for integration and segregation dynamics.

RANDOM AND SYSTEMATIC VARIATION IN SEGREGATION DYNAMICS

Discrete Choice Models

Our approach to segregation dynamics is to assume that people's evaluation of neighborhoods is consistent with a discrete choice model. In the model, an individual (or household) selects a neighborhood on the basis of its relative desirability. We typically observe only whether a particular neighborhood is chosen and not the actual desirability of that choice. If u_{i1} denotes the desirability (utility) of neighborhood 1, and u_{i2} denotes the utility of neighborhood 2 for person i , and person i selects neighborhood 1, then $u_{i1} > u_{i2}$. Although utility is unobserved, the effects of measured characteristics of neighborhoods that may affect their relative desirability can be estimated. The utility of the j th neighborhood for the i th individual is a combination of systematic (observable) and random (unobservable) components:

$$u_{ij} = \mu_{ij} + \varepsilon_{ij}, \quad (1)$$

where μ_{ij} is a (weighted) combination of observed characteristics of neighborhood j , possibly interacted with characteristics of person i , and ε_{ij} is a random error term. The random term includes unobserved characteristics of neighborhoods and heterogeneity among persons in how they evaluate neighborhoods.

The probability of choosing neighborhood k is the probability that the utility associated with k is greater than the utility associated with all other neighborhoods. If $y_{ik} = 1$ if neighborhood k is chosen and 0 otherwise,

$$\begin{aligned} \Pr(y_{ik} = 1) &= \Pr(u_{ik} > u_{ij} \text{ for all } j \neq k) \\ &= \Pr(\mu_{ik} + \varepsilon_{ik} > \mu_{ij} + \varepsilon_{ij} \text{ for all } j \neq k) \\ &= \Pr(\mu_{ik} - \mu_{ij} > \varepsilon_{ij} - \varepsilon_{ik} \text{ for all } j \neq k) \end{aligned} \quad (2)$$

Thus, the probability of choosing one neighborhood over another depends on the relative sizes of neighborhood differences in the systematic and random components of utility.

In a discrete choice model we cannot estimate the distribution of the random errors and thus, to identify the model, assume a fixed distribution for ε_{ij} , typically an extreme-value distribution. Given this assumption, one can estimate the coefficients for the observed variables that make up μ_{ij} and compute the probability of choosing each neighborhood using a conditional logit specification (McFadden 1974):

$$\Pr(y_{ij} = 1 | \mu_{ij}) = \frac{e^{\mu_{ij}}}{\sum_{k \in C_i} e^{\mu_{ik}}} \quad (3)$$

where $k \in C_i$ denotes the neighborhoods available to the i th individual.

Equation 2 shows that the probability of choosing one neighborhood over another neighborhood depends on the relative size of the systematic and random differences between the two neighborhoods. The distribution of random differences among neighborhoods, ε_{ij} , is fixed by assumption.² But the distribution of systematic differences among neighborhoods depends on the functional form and coefficient values of the choice function. Changing the function effectively changes the ratio of “signal to noise” in the choice process.

² An alternative way to identify the model is to assume a fixed distribution for the underlying utilities u_{ij} instead of the ε_{ij} (Winship and Mare 1983). This assumption results in a different scaling of the coefficients associated with the variables that contribute to μ_{ij} , but does not affect the relationships shown in Equations (2) and (3).

Systematic and Random Variation in the Schelling and Bruch-Mare Models

In Schelling's (1972, 1978) model, individuals prefer neighborhoods where the proportion of like neighbors is at least 50 percent. In this model the utility for the i th individual from the j th neighborhood is

$$u_{ij} = \beta X_{ij} \quad (4)$$

where $X_{ij} = 1$ when the j th neighborhood has at least 50% neighbors of like color and is 0 otherwise, and $\beta = 1$. The variance of the Schelling utility function is

$$\text{var}(u_{ij}) = \beta^2 \text{var}[X_{ij}] \quad (5)$$

which implies that variation in utility depends entirely on systematic variation in neighborhood proportion own-group.³ Bruch and Mare (2006) propose several alternatives to Schelling's original model, which are special cases of the discrete choice model shown in Equation 3 and are monotonic functions of the proportion of persons in an individual's own group in each neighborhood. The nonzero probability model is a threshold model in that individuals prefer areas that are at least 50% own group, although they have a nonzero probability of moving into any area. As with the Schelling model, individuals are indifferent to neighborhood changes that do not cross the 50% threshold. In this model the utility associated with the j th neighborhood is

$$u_{ij} = \beta X_{ij} + \varepsilon_{ij} \quad (6)$$

³ The Schelling model differs from the random utility models in several ways. First, it has no residual in the utility function. Although the actual choice of destination is probabilistic in that individuals randomly choose among acceptable destinations, these destinations are identical in their observed and unobserved characteristics. Second, the decision to leave one's current location is treated differently from the decision to move to a new destination. Individuals who are content in their current location, have zero probability of moving elsewhere, even for neighborhoods that exceed the threshold of 50% own group. Third, the function assigns a zero probability to all neighborhoods less than 50 percent own group. Agents never move to an undesirable neighborhood. If no desirable neighborhood is available, the individual remains in his or her current neighborhood until one is available.

where X_{ij} is 1 when the neighborhood is at least 50% own group, and 0 otherwise. In the linear continuous model, we allow individuals to respond to any small change in neighborhood composition. The utility associated with the linear continuous model is

$$u_{ij} = \beta q_{ij} + \varepsilon_{ij}, \quad (7)$$

where q_{ij} is the proportion own-group in the neighborhood. Because X_{ij} in Equation (6) is a function of q_{ij} , the proportion own-group in the neighborhood, we refer to both functions generally as $f(q_{ij})$.

The variance of the utilities across neighborhoods depends on their systematic and random variation. The more systematic variation among neighborhoods, the less likely that random error dominates the choice process. The variance of utility in the discrete choice model is

$$\text{var}(u_{ij}) = \beta^2 \text{var}[f(q_{ij})] + \text{var}(\varepsilon_{ij}) \quad (8)$$

Because the error variance is fixed in this model, a bigger variance on the first term (the systematic component) means a larger ratio of signal to noise when evaluating neighborhoods. Equation 8 shows that three factors that affect the ratio of systematic to random variation in the discrete choice models. First, an increase in β leads to an increase in systematic variation of the utilities. Second, the form of the function f affects the ratio of systematic to random variation because the variance of X_{ij} and q_{ij} differ for a given neighborhood composition. Third, as we demonstrate more fully below, the distribution of neighborhood proportion own-group in the city, q_{ij} , affects the signal to noise ratio because the $\text{var}[f(q_{ij})]$ term takes it as an argument.

1. Functional Form and the Baseline Ratio of Systematic to Random Variation

The higher the coefficient β in discrete choice models, *ceteris paribus*, the higher the ratio of systematic to random variation in the choice process. In the Schelling model, however, all variation is systematic. Because the Schelling model contains no random variation, segregation dynamics produced by this choice function do not vary with β . This is illustrated in Figure 1, which shows that the time path of the index of dissimilarity for the Schelling model does not depend on β .

[INSERT FIGURE 1 HERE]

In contrast, in the threshold and continuous discrete choice models systematic and unobserved characteristics of neighborhoods affect neighborhood desirability. Moreover, because of how neighborhood racial composition enters the utility and variance functions for a given β value and neighborhood composition, the variances of utility in the nonzero and continuous models also differ implying that the functions have different relative sizes of systematic and random components of variability. Table 1 shows the systematic variance of the choice functions for different neighborhood distributions of proportion own-group. We compute the systematic variation implied by the continuous and threshold function for five hypothetical communities, each of which comprises ten neighborhoods: (1) complete integration (all neighborhoods are 50% own-group); (2) complete segregation (all neighborhoods are 0% or 100% own-group); (3) uniform distribution of percent own-group (neighborhoods are 10,20,30,....,90,100 % own-group); (4) near segregation (neighborhoods are all 10% or 90% own-group); and (5) near integration (all neighborhoods are 40% or 60% own-group). Because the threshold function does not respond to changes in neighborhood composition that occur above and below the threshold, the variance of the threshold function is less sensitive to the distribution of proportion own-group. For both the continuous and threshold functions,

the systematic component is largest when neighborhoods are close to 0 or 100 percent own group. Where communities are well integrated, mobility decisions are almost completely determined by the random component of neighborhood desirability. With the exception of complete segregation and complete integration (where the continuous and threshold functions have the same systematic component), the threshold function always has a bigger systematic component of variance. The bottom panel of Table 1 shows that the relative amount of systematic variation for all functions varies directly with the size of β , but the differences in systematic variance across functional forms and neighborhood composition remain.⁴ For any β , the threshold function has a systematic variance component that is greater than or equal to the systematic variance component for the continuous function.

2. Functional Form and the Cumulation of Randomness

The second way that functional form affects the ratio of random to systematic variation in the choice process is through differences in how randomness builds or dampens out with time. We argued that continuous functions may lead to lower levels of segregation than threshold functions because the continuous function is sensitive to very small changes in neighborhood composition, thus creating a “cascade toward integration” (Bruch and Mare 2006: 692-24). Even a small number of individuals, who by chance move into areas with few own-group members, increase the desirability of those areas to

⁴ This demonstration assumes a fixed distribution of neighborhood characteristics. But proportion own-group in neighborhoods (and thus $\text{var}[f(q_{ij})]$) changes over time as a result of changing neighborhood composition. When many neighborhoods are approximately 50% own-group, the variance of q_{ij} is low for both the threshold and continuous functions and thus the contribution of systematic variation to individuals' choices is low. When segregation is high, the variance of all functions is higher, thereby placing greater weight on the systematic component (neighborhood proportion own-group) of residential mobility decisions.

members of their group, thereby increasing the probability that more own-group members subsequently move there. Threshold functions, in contrast, have large intervals of proportion own-group across which individuals are indifferent. Thus, a small number of individuals who move to an area with few own-group members are unlikely to increase the desirability of that area for future movers. This argument is correct as far as it goes, but incomplete because it ignores the effect of random variation in this process.

In the continuous function, this randomness cumulates whereas in the threshold function it does not. As discussed above, the ratio of systematic to random variation in the model depends on functional form, the effect of race composition β , and race composition itself q_{ij} . For simplicity, imagine a community with two neighborhoods, one that is 100% black and one that is 100% white. An unlikely move of a white person into the black neighborhood slightly increases the proportion white in that area. This decreases to a small degree the variation in q_{ij} (proportion own-group) across the two neighborhoods. The continuous function implies that individuals immediately respond to this decrease in the variance of q_{ij} , creating a corresponding reduction in the difference in utility between the minority and majority group neighborhood. In the next time step, because the observed difference in the systematic part of utilities between neighborhoods has decreased slightly, the extent to which randomness affects the choice process has correspondingly increased. This makes it more likely that another white person enters a majority black neighborhood, further decreasing the variance of q_{ij} . In the threshold model, individuals are insensitive to small changes in q_{ij} provided that the change does not occur at the threshold point. Thus, small deviations from racial homogeneity do not decrease the relevant systematic differences between neighborhoods.

[INSERT FIGURE 2 HERE]

Figure 2 shows segregation dynamics under the nonzero and continuous functions for different values of β . The extent to which randomness cumulates in the continuous model depends on β . Higher values of β produce higher levels of segregation for all functions. However, the threshold function is less responsive to increases in β than the continuous function because the value of the latter function is more affected by random perturbations in choices. The continuous function registers a change in utility for any change in neighborhood composition. When β is low, a few individuals may move to less desirable areas (with low probability), but this in turn affects the desirability of those areas in the next round, and this process cumulates in such a way as to create integration. However, when β is high, a few individuals may move to less desirable areas, but the difference in utilities among neighborhood types is sufficiently large to keep random perturbation from cumulating with time. The threshold function is less susceptible to randomness because a small change in neighborhood composition rarely registers a change in neighborhood desirability (except at the threshold point). As a result, random perturbations tend not to have a cumulative effect on segregation.

3. Functional Form and Speed of Convergence to Equilibrium Segregation

Functional form affects segregation dynamics in a third way when systematic factors dominate the choice process. For very large values of β , the contribution of random error to the choice process is negligible. That errors tend to “cumulate” in the continuous function is irrelevant, because the system has so little randomness. Both the continuous and the threshold functions can, with a sufficiently large β , sustain a high level of segregation. The two types of functions, however, differ in the speed at which an equilibrium level of segregation comes about. With little noise in the choice function, individuals who follow a continuous function steadily gravitate towards increasingly homogeneous neighborhoods. For example, a person in a 60% own-group neighborhood is attracted to an 80% own-group neighborhood. Individuals seek the highest proportion own-group neighborhoods that they can. The result is a steady path to segregation.

When individuals follow a threshold function, they are indifferent to neighborhoods above and below the threshold. Regardless of β , those who live in satisfactory neighborhoods (greater than 50% own-group) are as likely to select a less homogenous as compared to a more homogeneous neighborhood so long as the neighborhoods are within the greater than 50% own-group range. Individuals who already live in satisfactory neighborhoods do not, by their actions, contribute to further segregation. The only individuals who contribute to further segregation under the threshold function are those who try to escape unsatisfactory areas. When individuals leave an area where their group is the local minority in favor of an area where they are in the majority, they increase segregation. Once these individuals move to a majority group neighborhood, they too move among neighborhoods where their own group is the local majority.

This explains why, for higher values of β , the continuous function segregates more quickly than the threshold function. If individuals follow a continuous function, they gravitate to neighborhoods with ever higher percentages of persons in their own group; whereas if they follow a threshold function, they leave areas where they are the local minority, but move among all majority-group areas regardless of their exact racial makeup. In the threshold model, only individuals who leave areas where they are the local minority raise the level of segregation. Over time, a smaller and smaller number of individuals are in a position to increase segregation. Ultimately, all individuals who can leave an area where they are the local minority will do so, and (subject to availability constraints) this generates complete segregation in the threshold model. But it takes longer to get there in the threshold model than in the continuous model.

[INSERT FIGURE 3 HERE]

Figure 3 illustrates how, at high β values (where randomness plays a trivial role in residential choice) functional form affects the rate at which neighborhoods segregate. It shows segregation levels for the continuous and threshold functions for two values of β simulated over 20 million time ticks. When $\beta = 55$, the continuous function segregates faster than the threshold function, reaching maximum segregation at approximately 3 million ticks and stabilizing thereafter. Although we did not run the model to its exact equilibrium, the gap between the threshold and continuous functions shrinks steadily over time. Eventually, the two segregation lines converge. The dashed vertical line in Figure 3 marks the implied segregation levels at 1,000,000 ticks, the duration of VSM's simulations shown in their Figure C1. At that point, the implied segregation level at $\beta = 55$ is markedly higher for the continuous than the threshold function. But VSM's conclusion that the continuous function produces a higher segregation level is an artifact

of failing to run the simulations long enough. The two functions eventually converge to the same level of segregation, albeit at different rates.

Sensitivity to Chance or Sensitivity to Change?

VSM (p. 5) claim that that high randomness (“sensitivity to chance”) matters more than functional form assumptions (“sensitivity to change”). We have demonstrated the importance of functional form assumptions, even in conditions where there is little randomness. We agree with VSM that the β coefficient (the strength of preferences) plays a key role in segregation dynamics, and acknowledge the error on this point in our original analysis. However, it is misleading to treat β as “sensitivity to chance” (the degree to which randomness defines the choice process) and functional form as determining “sensitivity to change,” and to focus on a comparison of the relative sizes of these effects. Both factors affect the relationship between systematic and random variation in the model and do so in an interactive way. When the amount of randomness in the choice process is small, the continuous function leads to a much faster rate of segregation than the threshold function, but both functions produce a high level of segregation. VSM’s conclusion that continuous functions yield more segregation is an artifact of running their simulations for too few time steps.

Finally, both our analyses and those in VSM show that the threshold function is less responsive to changes in β . The threshold function is more robust to changes in the ratio of systematic to random variation in choices, both because the threshold function has less random variation relative to systematic variation to begin with, and also because randomness in the choice process does not cumulate in the threshold function. At low values of β , the threshold function leads to a much higher level of segregation than the

continuous function. For higher β values, the continuous function segregates faster than the threshold function.

PREFERENCES FOR INTEGRATION AND SEGREGATION DYNAMICS

The corrected results presented above show that continuous functions can generate segregation, even for relatively low β values. Figure 4 is a correction of Figure 6 from our 2006 paper and shows the levels of segregation implied by the preferences expressed through responses to neighborhood vignettes by respondents to the 1992 DAS. In that survey, whites expressed a monotonically increasing preference for living in areas with more whites, whereas blacks expressed a preference for racially mixed areas. The dotted line in the figure shows segregation levels implied by the race-specific responses. These empirically based preference functions are sufficient to generate a high level of segregation. It is useful to consider, however, whether whites alone are sustaining this high level of segregation. Figure 4 also shows simulation results under two hypothetical situations: (1) both blacks and whites have the ethnocentric own-group preferences demonstrated by Detroit whites (solid line), and (2) both blacks and whites have the preference for integration demonstrated by Detroit blacks (dashed line) (Bruch and Mare 2006, Table 2). If both blacks and whites had the own-group preferences of whites, a very high level of segregation would result. But if both blacks and whites held blacks' preference for mixed race neighborhoods, a very low level of segregation ($D \approx 0.1$) would result. In short, blacks' preferences for integration are consistent with a low level of segregation.

How do blacks' preferences for integration sustain segregation? The results shown in Figure 4 reflect not just a preference for diversity on the part of blacks but also

their willingness to tolerate a range of diverse neighborhoods that makes integrated neighborhoods resilient to small perturbations in racial makeup. Among DAS respondents, at least 85 percent of blacks were willing to live in neighborhoods between 29-86 percent white (Bruch and Mare 2006, Table 1). Whereas preferences for these neighborhoods vary somewhat, the empirical function approximates a “region of indifference” among neighborhoods with widely varying racial makeup. Our analysis of threshold functions reveals that regions of indifference in a preference function prevent random perturbations from destabilizing residential patterns. What remains unknown is how large a region of indifference (or relative indifference) is needed to sustain integration. This is a good topic for future study.

The “Paradox” of Strong vs. Weak Preferences

What accounts for the discrepancy between our empirical results about preference for integration and VSM’s simulation results, which, they argue, show that integrationist preferences lead to segregation? The answer lies in the different shapes of our empirical and their hypothetical preference functions. The DAS black respondents were relatively indifferent over a range of integrated neighborhoods. The DAS data suggest that African Americans would be satisfied with integrated areas between 40 and 60 percent own group. But this is in sharp contrast to the assumptions made in VSM’s “continuous” function (described in their footnote 8). Figure 5 plots the VSM continuous function for various values of β . Whereas individuals who follow this function do prefer integrated neighborhoods, only a neighborhood of *exactly* 50% own-group neighbors is fully satisfactory. In addition, all neighborhoods above the 50% own-group mark are considered more satisfactory than neighborhoods that are an equal percentage below

50%.⁵ The VSM function has an unusually sharp peak, which contains no stable range of heterogeneous neighborhoods, combined with an asymmetry that favors neighborhoods with a higher percentage own-group. This shape dictates VSM's results; to wit, that integrated neighborhoods are highly unstable and, *ceteris paribus*, individuals tend to choose neighborhoods with relatively more members of their own group. All persons have a strong incentive to move into the highly desirable 50% own-group neighborhood. Unfortunately, even one person moving into a perfectly integrated area immediately destabilizes that neighborhood. Ironically, for VSM, increasing the "strength of integrationist preferences" (P. XXXX) amounts to increasing the peakedness of the preference function, which makes living in a 50/50 or nearly 50/50 neighborhood harder to attain. VSM argue that increasing β increases the "strength" of multiethnic preferences, but, because of the asymmetry of the function, it actually increases the desirability of majority group areas relative to minority group areas.

VSM argue that there is a "paradox of strong versus weak preferences: when ethnic preferences are sufficiently weak relative to chance, sensitivity to change can lead to greater integration in a population that prefers segregation, and when ethnic preferences are sufficiently strong, sensitivity to change can lead to greater segregation in a population that prefers diversity" (p. XXXX). We see no paradox. The horse race between the "threshold" and "continuous" preference for integration is too stylized to be informative. VSM's continuous function does not lead to integration because the preference for integrated neighborhoods is so highly concentrated on 50/50 neighborhoods. Readers should be skeptical of the generality of VSM's result. Our

⁵ A 51 percent own group neighborhood is considered more desirable than a 49 percent own group neighborhood, a 55 percent own-group neighborhood is considered more desirable than a 45 percent own-group neighborhood, and so forth. This pattern is not evident in VSM's Figure C3, which does not represent the actual nonlinearity of the function.

function for DAS blacks, which would, if shared by both races, generate integration, provides some empirical grounds for skepticism.

CONCLUSION

We have corrected errors in our 2006 article and reassessed our prior claims in light of these errors. We acknowledge that realistic preferences can result in high levels of segregation, a change from our previous conclusion. We also acknowledge that monotonic continuous functions with a sufficiently high β can lead to segregation. However, our original analysis of the role of functional form (that is, how people respond to neighborhood characteristics) in segregation dynamics is correct, albeit incomplete. Functional form affects segregation dynamics: (1) via the baseline ratio of systematic to random variation in the choice process; (2) via whether or not randomness “cumulates” over time; and, (3) when random variation is small, via the time it takes to reach an equilibrium level of segregation. Because functional form and the strength of residential preferences interact in a complex way, it is misleading to focus on whether one or the other has a bigger effect on segregation. Although it was not the focus of our 2006 article, we have also examined how integrationist preferences affect segregation dynamics. If both blacks and whites had blacks’ preferences for integration, *ceteris paribus*, segregation would be low. Both continuous and threshold functions can lead to integration so long as individuals are relatively indifferent among a range of integrated neighborhoods. This ensures that integration is robust to small changes in neighborhood composition.

Our focus on technical issues relating to neighborhood preferences and segregation dynamics notwithstanding, these issues boil down to competing ideas about the rules that govern the behavior of individuals and the distribution of populations.

Threshold and continuous response functions may lead to different levels of segregation even when the average level of tolerance is the same. When preferences are mild ($\beta < 5$) the threshold function generates higher segregation than the continuous function. When preferences are strong ($\beta > 5$) both functions generate high segregation and the rate of convergence to segregation is faster for the continuous function. Functional form matters, at both low and high levels of randomness.

From the standpoint of the analyst, how much randomness enters into processes of residential choice depends on how completely one enumerates the systematic properties of neighborhoods, on the variation across individuals in how these properties are evaluated, and on features of the community. The ethnic makeup of a neighborhood is of great interest to social scientists, but it is only one of many characteristics that affect how attractive a place is to prospective residents. For example, neighborhoods may vary by air quality and proximity to the beach as well as ethnic makeup, all of which affect their relative desirability. If included in an analysis of residential choice, these characteristics contribute to systematic variation; if omitted, they contribute to randomness. Even if these characteristics are measured, individuals may vary in how much weight they place on these characteristics. To some extent we can capture differences in individual preferences with such simple rules of thumb as that blacks and whites place different (and opposite) weights on the percentage of neighborhood residents who are black. But this is an oversimplified representation of preference heterogeneity across individuals, who vary in their racial tolerance and tastes for neighborhood amenities. Unmeasured heterogeneity in preferences, therefore, contributes further random variation to the choice process. Threshold and continuous functions imply different responses to this heterogeneity in population and neighborhood

characteristics. Furthermore, communities with more diversity among people and neighborhoods may have different levels of segregation from areas with lower diversity. These effects of diversity depend on the behavioral rules that link neighborhood characteristics and individual actions.

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Table 1. Variance of Nonzero (threshold) and Continuous Function for 10 Neighborhoods

Neighborhoods				
	Mean	Variance	Min	Max
Integrated	0.5	0.000	0.5	0.5
Segregated	0.5	0.278	0	1
Uniform	0.55	0.092	0.1	1
Almost Segregated	0.5	0.225	0.05	0.95
Almost Integrated	0.5	0.003	0.45	0.55

Threshold Utilities				
	Mean	Variance	Min	Max
$\beta = 1$				
Integrated	1.0	0.000	1	1
Segregated	0.5	0.278	0	1
Uniform	0.6	0.267	0	1
Almost Segregated	0.5	0.278	0	1
Almost Integrated	0.5	0.278	0	1
$\beta = 2$				
Integrated	2.0	0.000	2	2
Segregated	1.0	1.111	0	2
Uniform	1.2	1.067	0	2
Almost Segregated	1.0	1.111	0	2
Almost Integrated	1.0	1.111	0	2

Continuous Utilities				
	Mean	Variance	Min	Max
$\beta = 1$				
Integrated	0.5	0.000	0.5	0.5
Segregated	0.5	0.278	0	1
Uniform	0.55	0.092	0.1	1
Almost Segregated	0.5	0.225	0.05	0.95
Almost Integrated	0.5	0.003	0.45	0.55
$\beta = 2$				
Integrated	1	0.000	1	1
Segregated	1	1.111	0	2
Uniform	1.1	0.367	0.2	2
Almost Segregated	1	0.900	0.1	1.9
Almost Integrated	1	0.011	0.9	1.1

Figure 1. Schelling Model with Varying Betas

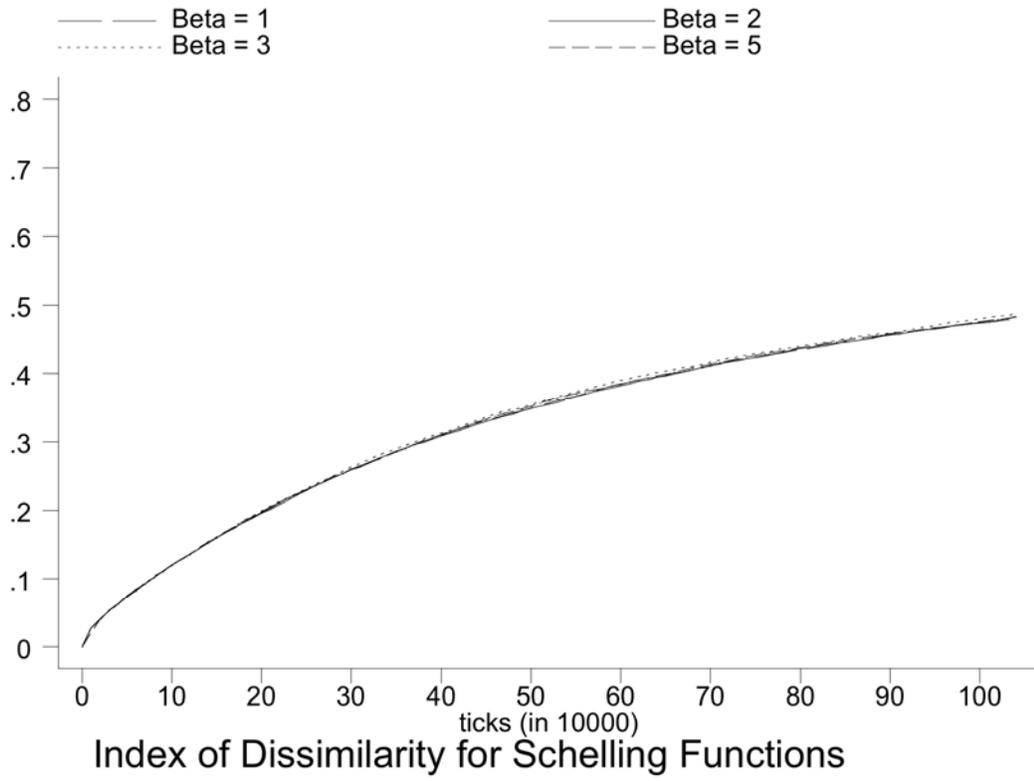


Figure 2. Segregation Outcomes for Nonzero (threshold) and Continuous Functions, Varying Beta, 1 million iterations

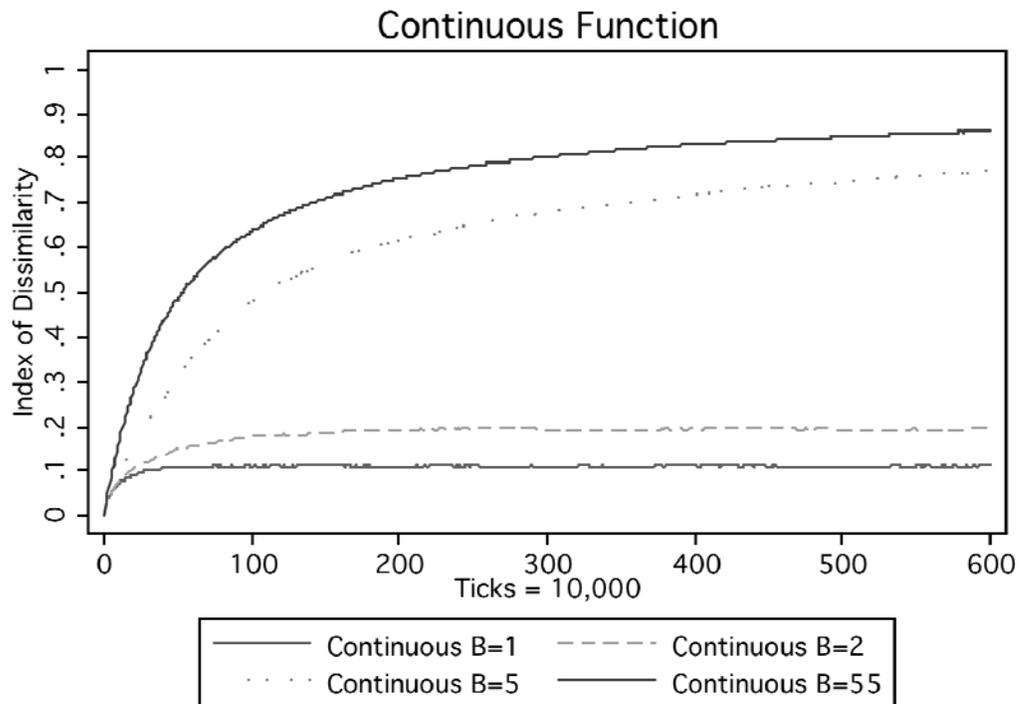
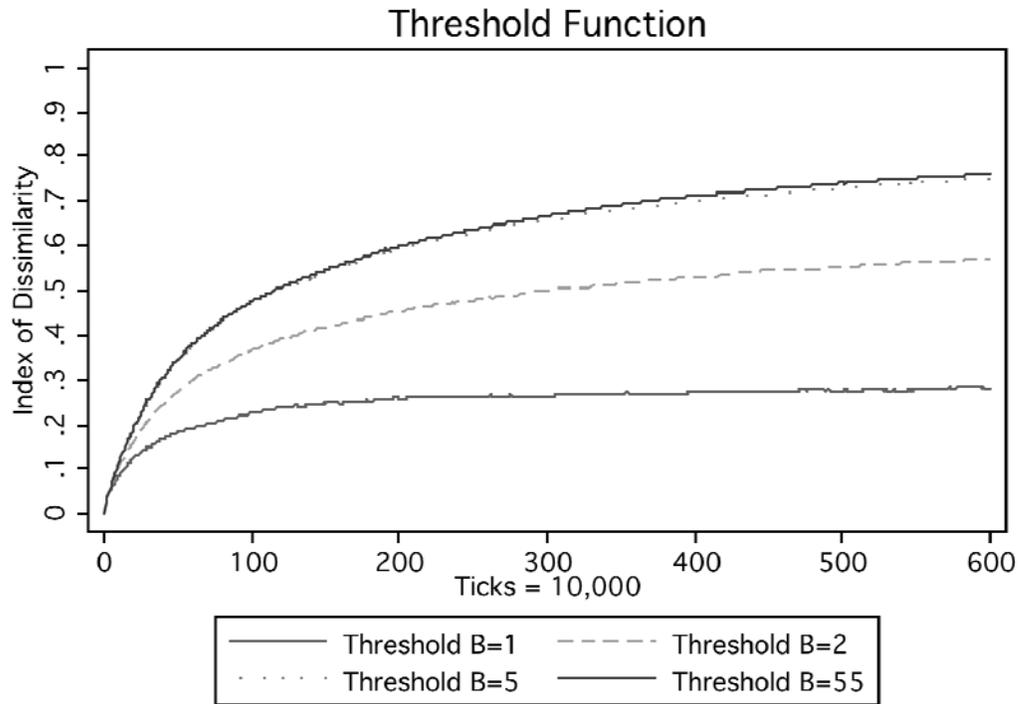


Figure 3. Illustration of Convergence in Threshold and Continuous Functions for High Beta Values, 15 million iterations

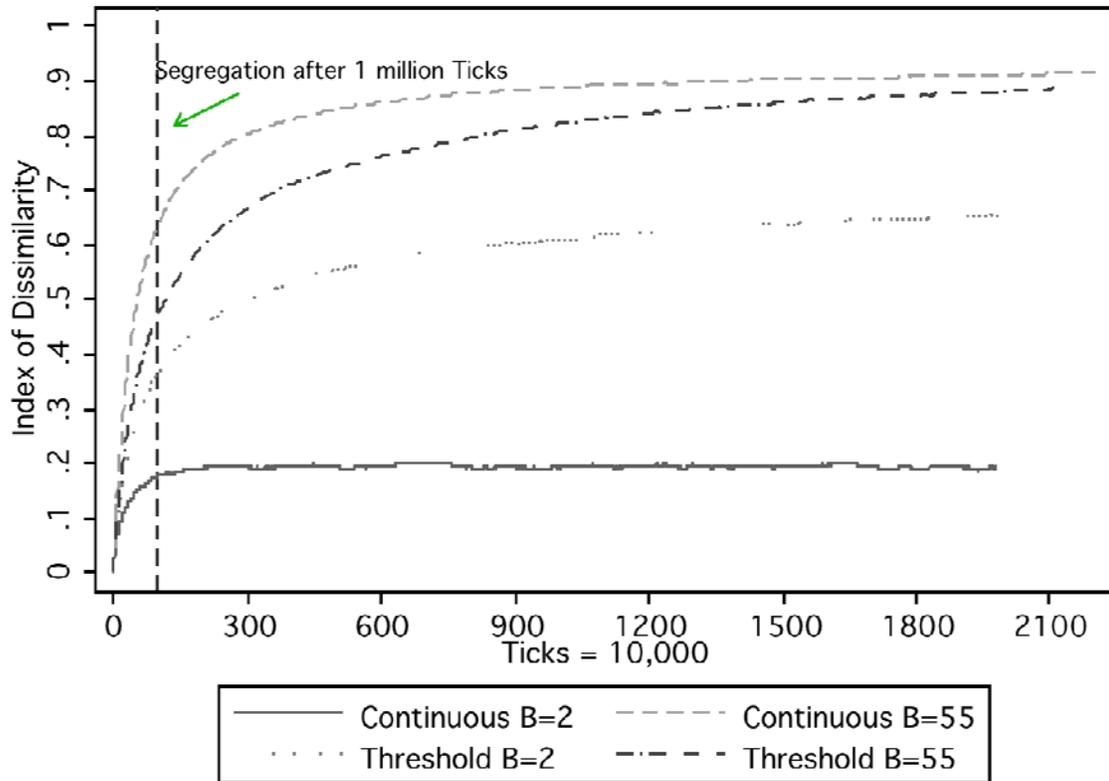


Figure 4. Segregation Outcomes, DAS preferences, and hypothetical preferences

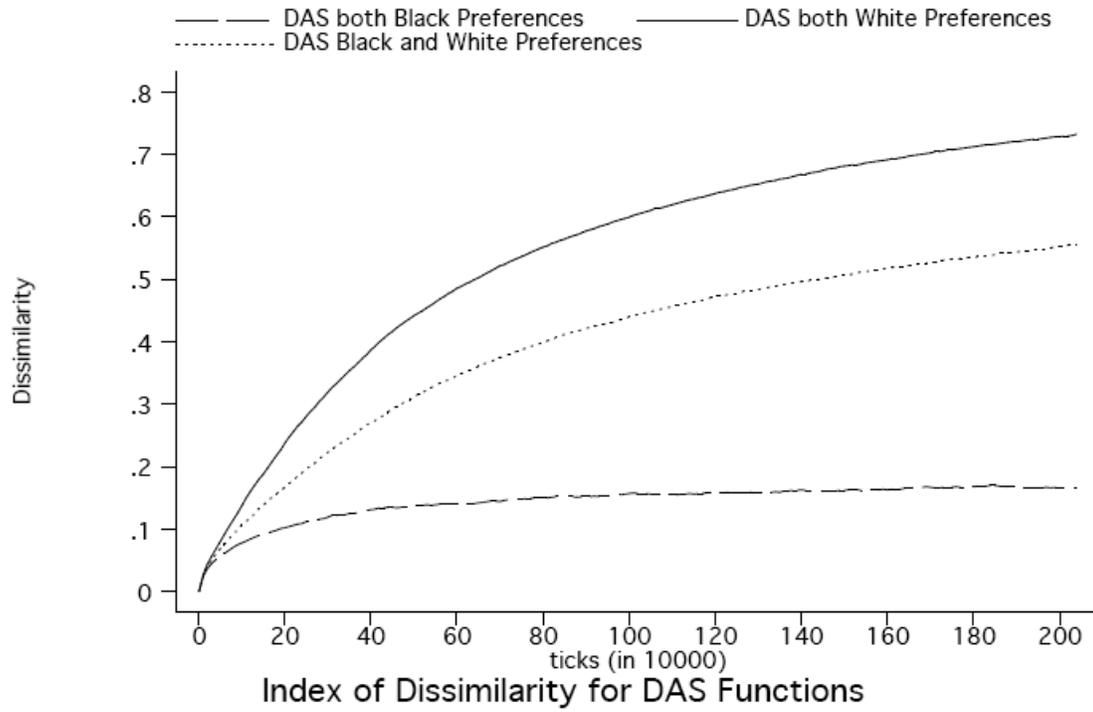
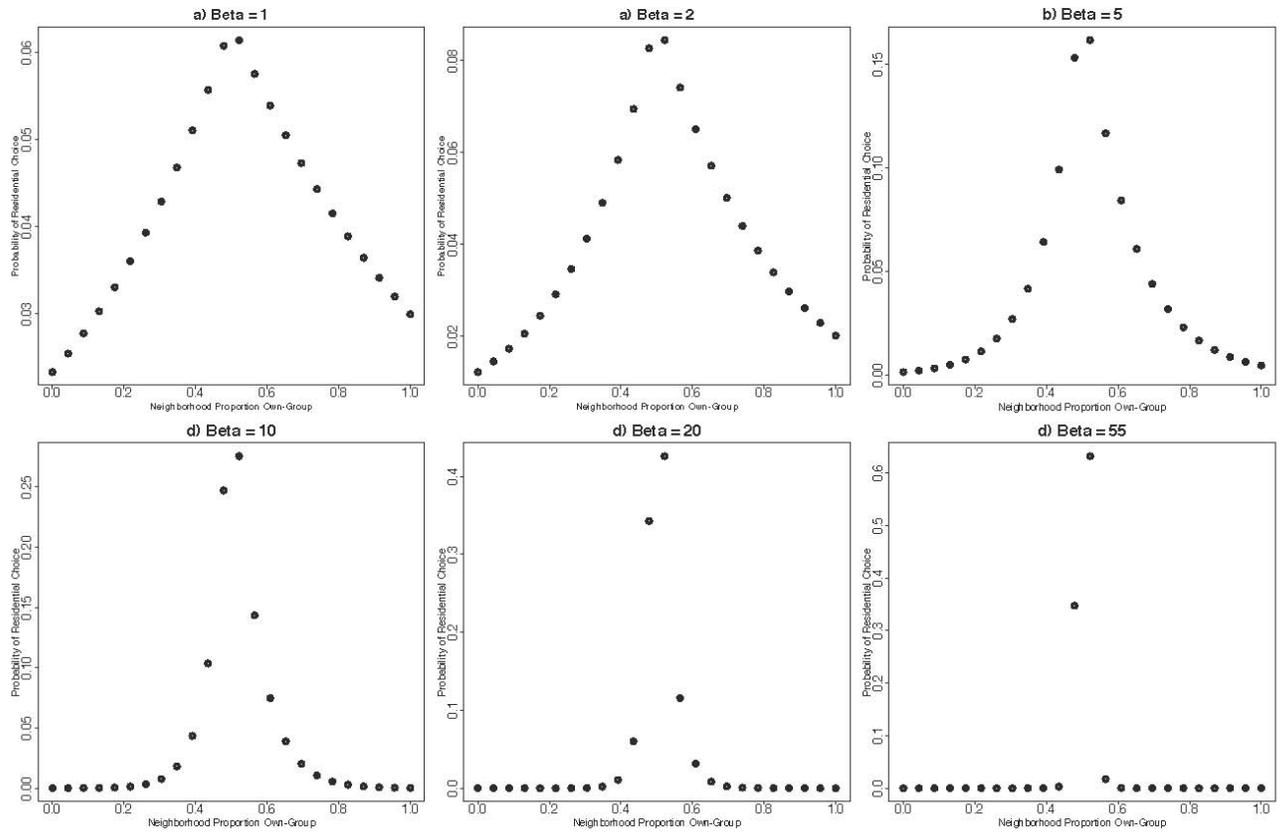


Figure 5. Illustrations of Van de Rijt, Macy, and Siegel (2008) Continuous “Multiculturalists” Preference Function¹



¹ Note that the interval of proportion own group is divided into 24 intervals, to correspond to the size of the neighborhoods in our agent-based models. Each neighborhood can hold a maximum of 25 agents.